Patterns of Sums & Products of Decimal Digits

#83 of Gottschalk's Gestalts

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GG83-2

□ here are some patterns
that are formed from
systematic sums & products
of the ten decimal digits
0, 1, 2, 3, 4, 5, 6, 7, 8, 9
plus
some easily accessible topics
in the elementary theory of numbers
that flow from these patterns

☐ the addition table
for the ten decimal digits

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

□ the ten rows & the ten columns
of the sums of the addition table
constitute arithmetic progressions;
these twenty components of the addition table
are displayed separately below

• the symmetry of the matrix of the addition table
reflects the commutativity of addition

• observe the properties of the diagonals & antidiagonals;
the diagonals are all arithmetic progressions;
the antidiagonals are all constant sequences

• sums with 0 as summand

$$0+0 = 0 = 0+0$$
$$0+1 = 1 = 1+0$$
$$0+2 = 2 = 2+0$$
$$0+3 = 3 = 3+0$$
$$0+4 = 4 = 4+0$$
$$0+5 = 5 = 5+0$$
$$0+6 = 6 = 6+0$$
$$0+7 = 7 = 7+0$$
$$0+8 = 8 = 8+0$$
$$0+9 = 9 = 9+0$$

the entries in the central column
form an arithmetic progression with
the number of terms = 10
the first term = 0
the last term = 9
the common difference = 1

• sums with 1 as summand

$$1+0 = 1 = 0+1$$
$$1+1 = 2 = 1+1$$
$$1+2 = 3 = 2+1$$
$$1+3 = 4 = 3+1$$
$$1+4 = 5 = 4+1$$
$$1+5 = 6 = 5+1$$
$$1+6 = 7 = 6+1$$
$$1+7 = 8 = 7+1$$
$$1+8 = 9 = 8+1$$
$$1+9 = 10 = 9+1$$

the entries in the central column
form an arithmetic progression with
the number of terms = 10
the first term = 1
the last term = 10
the common difference = 1

• sums with 2 as summand

$$2+0 = 2 = 0+2$$
$$2+1 = 3 = 1+2$$
$$2+2 = 4 = 2+2$$
$$2+3 = 5 = 3+2$$
$$2+4 = 6 = 4+2$$
$$2+5 = 7 = 5+2$$
$$2+6 = 8 = 6+2$$
$$2+7 = 9 = 7+2$$
$$2+8 = 10 = 8+2$$
$$2+9 = 11 = 9+2$$

the entries in the central column
form an arithmetic progression with
the number of terms = 10
the first term = 2
the last term = 11
the common difference = 1

• sums with 3 as summand

$$
\begin{aligned}
3+0 &= 3 = 0+3\\
3+1 &= 4 = 1+3\\
3+2 &= 5 = 2+3\\
3+3 &= 6 = 3+3\\
3+4 &= 7 = 4+3\\
3+5 &= 8 = 5+3\\
3+6 &= 9 = 6+3\\
3+7 &= 10 = 7+3\\
3+8 &= 11 = 8+3\\
3+9 &= 12 = 9+3
\end{aligned}
$$

the entries in the central column
form an arithmetic progression with
the number of terms $= 10$
the first term $= 3$
the last term $= 12$
the common difference $= 1$

• sums with 4 as summand

$$
\begin{aligned}
4+0 &= 4 = 0+4 \\
4+1 &= 5 = 1+4 \\
4+2 &= 6 = 2+4 \\
4+3 &= 7 = 3+4 \\
4+4 &= 8 = 4+4 \\
4+5 &= 9 = 5+4 \\
4+6 &= 10 = 6+4 \\
4+7 &= 11 = 7+4 \\
4+8 &= 12 = 8+4 \\
4+9 &= 13 = 9+4
\end{aligned}
$$

the entries in the central column
form an arithmetic progression with
the number of terms = 10
the first term = 4
the last term = 13
the common difference = 1

• sums with 5 as summand

$$5+0 = 5 = 0+5$$
$$5+1 = 6 = 1+5$$
$$5+2 = 7 = 2+5$$
$$5+3 = 8 = 3+5$$
$$5+4 = 9 = 4+5$$
$$5+5 = 10 = 5+5$$
$$5+6 = 11 = 6+5$$
$$5+7 = 12 = 7+5$$
$$5+8 = 13 = 8+5$$
$$5+9 = 14 = 9+5$$

the entries in the central column
form an arithmetic progression with
the number of terms = 10
the first term = 5
the last term = 14
the common difference = 1

• sums with 6 as summand

$$6+0 = 6 = 0+6$$
$$6+1 = 7 = 1+6$$
$$6+2 = 8 = 2+6$$
$$6+3 = 9 = 3+6$$
$$6+4 = 10 = 4+6$$
$$6+5 = 11 = 5+6$$
$$6+6 = 12 = 6+6$$
$$6+7 = 13 = 7+6$$
$$6+8 = 14 = 8+6$$
$$6+9 = 15 = 9+6$$

the entries in the central column
form an arithmetic progression with
the number of terms = 10
the first term = 6
the last term = 15
the common difference = 1

• sums with 7 as summand

$$
\begin{aligned}
7+0 &= 7 = 0+7\\
7+1 &= 8 = 1+7\\
7+2 &= 9 = 2+7\\
7+3 &= 10 = 3+7\\
7+4 &= 11 = 4+7\\
7+5 &= 12 = 5+7\\
7+6 &= 13 = 6+7\\
7+7 &= 14 = 7+7\\
7+8 &= 15 = 8+7\\
7+9 &= 16 = 9+7
\end{aligned}
$$

the entries in the central column
form an arithmetic progression with
the number of terms = 10
the first term = 7
the last term = 16
the common difference = 1

• sums with 8 as summand

$8+0 = 8 = 0+8$

$8+1 = 9 = 1+8$

$8+2 = 10 = 2+8$

$8+3 = 11 = 3+8$

$8+4 = 12 = 4+8$

$8+5 = 13 = 5+8$

$8+6 = 14 = 6+8$

$8+7 = 15 = 7+8$

$8+8 = 16 = 8+8$

$8+9 = 17 = 9+8$

the entries in the central column
form an arithmetic progression with
the number of terms = 10
the first term = 8
the last term = 17
the common difference = 1

• sums with 9 as summand

$$
\begin{aligned}
9+0 &= 9 = 0+9\\
9+1 &= 10 = 1+9\\
9+2 &= 11 = 2+9\\
9+3 &= 12 = 3+9\\
9+4 &= 13 = 4+9\\
9+5 &= 14 = 5+9\\
9+6 &= 15 = 6+9\\
9+7 &= 16 = 7+9\\
9+8 &= 17 = 8+9\\
9+9 &= 18 = 9+9
\end{aligned}
$$

the entries in the central column
form an arithmetic progression with
the number of terms = 10
the first term = 9
the last term = 18
the common difference = 1

☐ the multiplication table
for the ten decimal digits

X	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

□ the ten rows & the ten columns
of the products of the multiplication table
constitute arithmetic progressions;
these twenty components of the multiplication table
are displayed separately below

• the symmetry of the matrix of the multiplication table
reflects the commutativity of multiplication

• observe the properties of the diagonals & antidiagonals;
eg the main diagonal is the sequnce of perfect squares

- products with 0 as factor

$0 \times 0 = 0 = 0 \times 0$

$0 \times 1 = 0 = 1 \times 0$

$0 \times 2 = 0 = 2 \times 0$

$0 \times 3 = 0 = 3 \times 0$

$0 \times 4 = 0 = 4 \times 0$

$0 \times 5 = 0 = 5 \times 0$

$0 \times 6 = 0 = 6 \times 0$

$0 \times 7 = 0 = 7 \times 0$

$0 \times 8 = 0 = 8 \times 0$

$0 \times 9 = 0 = 9 \times 0$

the entries in the central column
form an arithmetic progression with
the number of terms = 10
the first term = 0
the last term = 0
the common difference = 0

• products with 1 as factor

$1 \times 0 = 0 = 0 \times 1$

$1 \times 1 = 1 = 1 \times 1$

$1 \times 2 = 2 = 2 \times 1$

$1 \times 3 = 3 = 3 \times 1$

$1 \times 4 = 4 = 4 \times 1$

$1 \times 5 = 5 = 5 \times 1$

$1 \times 6 = 6 = 6 \times 1$

$1 \times 7 = 7 = 7 \times 1$

$1 \times 8 = 8 = 8 \times 1$

$1 \times 9 = 9 = 9 \times 1$

the entries in the central column
form an arithmetic progression with
the number of terms = 10
the first term = 0
the last term = 9
the common difference = 1

• products with 2 as factor

$$2\times0 = 0 = 0\times2$$
$$2\times1 = 2 = 1\times2$$
$$2\times2 = 4 = 2\times2$$
$$2\times3 = 6 = 3\times2$$
$$2\times4 = 8 = 4\times2$$
$$2\times5 = 10 = 5\times2$$
$$2\times6 = 12 = 6\times2$$
$$2\times7 = 14 = 7\times2$$
$$2\times8 = 16 = 8\times2$$
$$2\times9 = 18 = 9\times2$$

the entries in the central column
form an arithmetic progression with
the number of terms = 10
the first term = 0
the last term = 18
the common difference = 2

• products with 3 as factor

$3\times0 = 0 = 0\times3$

$3\times1 = 3 = 1\times3$

$3\times2 = 6 = 2\times3$

$3\times3 = 9 = 3\times3$

$3\times4 = 12 = 4\times3$

$3\times5 = 15 = 5\times3$

$3\times6 = 18 = 6\times3$

$3\times7 = 21 = 7\times3$

$3\times8 = 24 = 8\times3$

$3\times9 = 27 = 9\times3$

the entries in the central column
form an arithmetic progression with
the number of terms = 10
the first term = 0
the last term = 27
the common difference = 3

• products with 4 as factor

$$4 \times 0 = 0 = 0 \times 4$$
$$4 \times 1 = 4 = 1 \times 4$$
$$4 \times 2 = 8 = 2 \times 4$$
$$4 \times 3 = 12 = 3 \times 4$$
$$4 \times 4 = 16 = 4 \times 4$$
$$4 \times 5 = 20 = 5 \times 4$$
$$4 \times 6 = 24 = 6 \times 4$$
$$4 \times 7 = 28 = 7 \times 4$$
$$4 \times 8 = 32 = 8 \times 4$$
$$4 \times 9 = 36 = 9 \times 4$$

the entries in the central column
form an arithmetic progression with
the number of terms = 10
the first term = 0
the last term = 36
the common difference = 4

• products with 5 as factor

$5 \times 0 = 0 = 0 \times 5$

$5 \times 1 = 5 = 1 \times 5$

$5 \times 2 = 10 = 2 \times 5$

$5 \times 3 = 15 = 3 \times 5$

$5 \times 4 = 20 = 4 \times 5$

$5 \times 5 = 25 = 5 \times 5$

$5 \times 6 = 30 = 6 \times 5$

$5 \times 7 = 35 = 7 \times 5$

$5 \times 8 = 40 = 8 \times 5$

$5 \times 9 = 45 = 9 \times 5$

the entries in the central column
form an arithmetic progression with
the number of terms = 10
the first term = 0
the last term = 45
the common difference = 5

• products with 6 as factor

$$6 \times 0 = 0 = 0 \times 6$$
$$6 \times 1 = 6 = 1 \times 6$$
$$6 \times 2 = 12 = 2 \times 6$$
$$6 \times 3 = 18 = 3 \times 6$$
$$6 \times 4 = 24 = 4 \times 6$$
$$6 \times 5 = 30 = 5 \times 6$$
$$6 \times 6 = 36 = 6 \times 6$$
$$6 \times 7 = 42 = 7 \times 6$$
$$6 \times 8 = 48 = 8 \times 6$$
$$6 \times 9 = 54 = 9 \times 6$$

the entries in the central column
form an arithmetic progression with
the number of terms = 10
the first term = 0
the last term = 54
the common difference = 6

• products with 7 as factor

$$7 \times 0 = 0 = 0 \times 7$$
$$7 \times 1 = 7 = 1 \times 7$$
$$7 \times 2 = 14 = 2 \times 7$$
$$7 \times 3 = 21 = 3 \times 7$$
$$7 \times 4 = 28 = 4 \times 7$$
$$7 \times 5 = 35 = 5 \times 7$$
$$7 \times 6 = 42 = 6 \times 7$$
$$7 \times 7 = 49 = 7 \times 7$$
$$7 \times 8 = 56 = 8 \times 7$$
$$7 \times 9 = 63 = 9 \times 7$$

the entries in the central column
form an arithmetic progression with
the number of terms $= 10$
the first term $= 0$
the last term $= 63$
the common difference $= 7$

• products with 8 as factor

$8 \times 0 = 0 = 0 \times 8$

$8 \times 1 = 8 = 1 \times 8$

$8 \times 2 = 16 = 2 \times 8$

$8 \times 3 = 24 = 3 \times 8$

$8 \times 4 = 32 = 4 \times 8$

$8 \times 5 = 40 = 5 \times 8$

$8 \times 6 = 48 = 6 \times 8$

$8 \times 7 = 56 = 7 \times 8$

$8 \times 8 = 64 = 8 \times 8$

$8 \times 9 = 72 = 9 \times 8$

the entries in the central column
form an arithmetic progression with
the number of terms = 10
the first term = 0
the last term = 72
the common difference = 8

• products with 9 as factor

$9 \times 0 = 0 = 0 \times 9$

$9 \times 1 = 9 = 1 \times 9$

$9 \times 2 = 18 = 2 \times 9$

$9 \times 3 = 27 = 3 \times 9$

$9 \times 4 = 36 = 4 \times 9$

$9 \times 5 = 45 = 5 \times 9$

$9 \times 6 = 54 = 6 \times 9$

$9 \times 7 = 63 = 7 \times 9$

$9 \times 8 = 72 = 8 \times 9$

$9 \times 9 = 81 = 9 \times 9$

the entries in the central column
form an arithmetic progression with
the number of terms = 10
the first term = 0
the last term = 81
the common difference = 9

□ in the following
there are several notions about integers
that apply in particular to decimal digits

□ for each natural number n
there are n +1 additive binary decompositions of n;
this fact is illustrated below
for the first twelve natural numbers n

• the single additive binary decomposition of 0

$0 = 0 + 0$

• the two additive binary decompositions of 1

$$1 = 0 + 1$$
$$1 = 1 + 0$$

the two columns of first and second summands
are inverted forms of each other,
the first two natural numbers in canonical order

• the three additive binary decompositions of 2

$$2 = 0 + 2$$
$$2 = 1 + 1$$
$$2 = 2 + 0$$

the two columns of first and second summands
are inverted forms of each other,
the first three natural numbers in canonical order

• the four additive binary decompositions of 3

$$3 = 0 + 3$$
$$3 = 1 + 2$$
$$3 = 2 + 1$$
$$3 = 3 + 0$$

the two columns of first and second summands
are inverted forms of each other,
the first four natural numbers in canonical order

• the five additive binary decompositions of 4

$$4 = 0 + 4$$
$$4 = 1 + 3$$
$$4 = 2 + 2$$
$$4 = 3 + 1$$
$$4 = 4 + 0$$

the two columns of first and second summands
are inverted forms of each other,
the first five natural numbers in canonical order

• the six additive binary decompositions of 5

$$5 = 0 + 5$$
$$5 = 1 + 4$$
$$5 = 2 + 3$$
$$5 = 3 + 2$$
$$5 = 4 + 1$$
$$5 = 5 + 0$$

the two columns of first and second summands
are inverted forms of each other,
the first six natural numbers in canonical order

• the seven additive binary decompositions of 6

$$6 = 0 + 6$$
$$6 = 1 + 5$$
$$6 = 2 + 4$$
$$6 = 3 + 3$$
$$6 = 4 + 2$$
$$6 = 5 + 1$$
$$6 = 6 + 0$$

the two columns of first and second summands
are inverted forms of each other,
the first seven natural numbers in canonical order

• the eight additive binary decompositions of 7

$7 = 0 + 7$

$7 = 1 + 6$

$7 = 2 + 5$

$7 = 3 + 4$

$7 = 4 + 3$

$7 = 5 + 2$

$7 = 6 + 1$

$7 = 7 + 0$

the two columns of first and second summands
are inverted forms of each other,
the first eight natural numbers in canonical order

• the nine additive binary decompositions of 8

$$
\begin{aligned}
8 &= 0+8\\
8 &= 1+7\\
8 &= 2+6\\
8 &= 3+5\\
8 &= 4+4\\
8 &= 5+3\\
8 &= 6+2\\
8 &= 7+1\\
8 &= 8+0
\end{aligned}
$$

the two columns of first and second summands
are inverted forms of each other,
the first nine natural numbers in canonical order

• the ten additive binary decompositions of 9

$9 = 0 + 9$

$9 = 1 + 8$

$9 = 2 + 7$

$9 = 3 + 6$

$9 = 4 + 5$

$9 = 5 + 4$

$9 = 6 + 3$

$9 = 7 + 2$

$9 = 8 + 1$

$9 = 9 + 0$

the two columns of first and second summands
are inverted forms of each other,
the first ten natural numbers in canonical order

• the eleven additive binary decompositions of 10

$10 = 0 + 10$

$10 = 1 + 9$

$10 = 2 + 8$

$10 = 3 + 7$

$10 = 4 + 6$

$10 = 5 + 5$

$10 = 6 + 4$

$10 = 7 + 3$

$10 = 8 + 2$

$10 = 9 + 1$

$10 = 10 + 0$

the two columns of first and second summands
are inverted forms of each other,
the first eleven natural numbers in canonical order

• the twelve additive binary decompositions of 11

$11 = 0 + 11$

$11 = 1 + 10$

$11 = 2 + 9$

$11 = 3 + 8$

$11 = 4 + 7$

$11 = 5 + 6$

$11 = 6 + 5$

$11 = 7 + 4$

$11 = 8 + 3$

$11 = 9 + 2$

$11 = 10 + 1$

$11 = 11 + 0$

the two columns of first and second summands
are inverted forms of each other,
the first twelve natural numbers in canonical order

☐ consecutive odd positive integer sums

- $1 = 1^2$
- $1+3 = 4 = 2^2$
- $1+3+5 = 9 = 3^3$
- $1+3+5+7 = 16 = 4^2$
- $1+3+5+7+9 = 25 = 5^2$
- $1+3+5+7+9+11 = 36 = 6^2$

etc

☐ consecutive even positive integer sums

- $2 = 1 \times 2$
- $2+4 = 6 = 2 \times 3$
- $2+4+6 = 12 = 3 \times 4$
- $2+4+6+8 = 20 = 4 \times 5$
- $2+4+6+8+10 = 30 = 5 \times 6$
- $2+4+6+8+10+12 = 42 = 6 \times 7$

etc

☐ consecutive positive integer sums

- $1 = \frac{1}{2} \times 1 \times (1+1)$
- $1+2 = 3 = \frac{1}{2} \times 2 \times (1+2)$
- $1+2+3 = 6 = \frac{1}{2} \times 3 \times (1+3)$
- $1+2+3+4 = 10 = \frac{1}{2} \times 4 \times (1+4)$
- $1+2+3+4+5 = 15 = \frac{1}{2} \times 5 \times (1+5)$
- $1+2+3+4+5+6 = 21 = \frac{1}{2} \times 6 \times (1+6)$

etc

☐ some uphill - and - downhill palindrome sums

- $1 = 1^2$
- $1+2+1 = 4 = 2^2$
- $1+2+3+2+1 = 9 = 3^2$
- $1+2+3+4+3+2+1 = 16 = 4^2$
- $1+2+3+4+5+4+3+2+1 = 25 = 5^2$
- $1+2+3+4+5+6+5+4+3+2+1 = 36 = 6^2$

etc

☐ these sums of cubes are squares

- $1^3 = 1 = \left(\frac{1}{2}\times 1\times 2\right)^2 = \frac{1}{4}\times 1^2\times 2^2$

- $1^3+2^3 = 9 = \left(\frac{1}{2}\times 2\times 3\right)^2 = \frac{1}{4}\times 2^2\times 3^2$

- $1^3+2^3+3^3 = 36 = \left(\frac{1}{2}\times 3\times 4\right)^2 = \frac{1}{4}\times 3^2\times 4^2$

- $1^3+2^3+3^3+4^3 = 100 = \left(\frac{1}{2}\times 4\times 5\right)^2 = \frac{1}{4}\times 4^2\times 5^2$

etc

D. partitions of positive integers

let

• n $\in$ pos int

then

• a partition of n

$=_{df}$ a weakly decreasing sequence of positive integers whose sum is n & which is usually written as an indicated sum

• the partition number of n

$=_{dn}$ p(n) wh p $\leftarrow$ partition

$=_{df}$ the number of partitions of n

☐ here are the partition numbers and some partitions of the first ten positive integers

• $p(1) = 1$

• $p(2) = 2$

since

2

$= 1+1$

• $p(3) = 3$

since

3

$= 2+1$

$= 1+1+1$

• $p(4) = 5$

since

$$
\begin{aligned}
& 4 \\
= \ & 3+1 \\
= \ & 2+2 \\
= \ & 2+1+1 \\
= \ & 1+1+1+1
\end{aligned}
$$

• $p(5) = 7$

since

$$
\begin{aligned}
& 5 \\
= \ & 4+1 \\
= \ & 3+2 \\
= \ & 3+1+1 \\
= \ & 2+2+1 \\
= \ & 2+1+1+1 \\
= \ & 1+1+1+1+1
\end{aligned}
$$

- $p(6) = 11$

since

$$
\begin{aligned}
& 6 \\
=\ & 5+1 \\
=\ & 4+2 \\
=\ & 4+1+1 \\
=\ & 3+3 \\
=\ & 3+2+1 \\
=\ & 3+1+1+1 \\
=\ & 2+2+2 \\
=\ & 2+2+1+1 \\
=\ & 2+1+1+1+1 \\
=\ & 1+1+1+1+1+1
\end{aligned}
$$

- $p(7) = 15$
- $p(8) = 22$
- $p(9) = 30$
- $p(10) = 42$
- $p(11) = 56$
- $p(12) = 77$
- $p(13) = 101$
- $p(14) = 135$
- $p(15) = 176$
- $p(100) = 190569292$

□ the successor function
going from n to n +1
where n is a natural number
generates
all natural numbers
starting with n = 0
eg
0 +1 = 1
1 +1 = 2
2 +1 = 3
3 +1 = 4
4 +1 = 5
5 +1 = 6
6 +1 = 7
7 +1 = 8
8 +1 = 9
9 +1 = 10
10 +1 = 11
11 +1 = 12
etc

D. factorials

let

- $n \in$ pos int

then

- the factorial of n

$=_{dn}$ n!

$=_{rd}$ n factorial = factorial n

$=_{df}$ the product of all n integers
from 1 to n inclusive

$= 1 \times 2 \times 3 \times \cdots \times n$

R. in order to make the identity

$(n+1)! = n! \times (n+1)$ wh $n \in$ pos int

also valid for $n = 0$,

define

the factorial of 0

to be

$0! = 1$

☐ factorials from 0! to 10!

$0! = 0$

$1! = 1$

$2! = 1\times 2 = 2$

$3! = 1\times 2\times 3 = 6$

$4! = 1\times 2\times 3\times 4 = 24$

$5! = 1\times 2\times 3\times 4\times 5 = 120$

$6! = 1\times 2\times 3\times 4\times 5\times 6 = 720$

$7! = 1\times 2\times 3\times 4\times 5\times 6\times 7 = 5040$

$8! = 1\times 2\times 3\times 4\times 5\times 6\times 7\times 8 = 40320$

$9! = 1\times 2\times 3\times 4\times 5\times 6\times 7\times 8\times 9 = 362880$

$10! = 1\times 2\times 3\times 4\times 5\times 6\times 7\times 8\times 9\times 10 = 3628800$

□ prime properties of prime numbers

• a positive integer
is said to be
a prime number
or
a prime (noun)
or
prime (adjective)
iff
the integer has exactly two divisors
viz
itself and 1

• the sequence of prime numbers
begins
$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, \cdots$

• there are infinitely many prime numbers

• a positive integer
is said to be
a composite number
or
a composite (noun)
or
composite (adjective)
iff
the integer is not a prime number

• the sequence of composite numbers begins
$1, 4, 6, 8, 9, 10, 12, 14, 15, 16, \cdots$

• there are infinitely many composite numbers

• if n is an integer ≥ 2,
then the following sequence
of $n-1$ consecutive positive integers
contains only composite numbers:
$n!+2, n!+3, n!+4, \cdots, n!+n$

• 2 is the only even prime;
all other primes are odd

• a plural integer n is prime
iff
every prime number $\leq \sqrt{n}$
does not divide n

• it is difficult to construct large prime numbers;
as of the year 2002
the largest known prime is
the Mersenne prime $2^n - 1$
where $n = 13466917$
and which has $4,053,946$ digits;
a Mersenne number is defined to be
an integer of the form $2^n - 1$
wh $n \in$ nonneg int

• an integer > 2 that ends in
one of the digits
$0, 2, 4, 5, 6, 8$
is necessarily composite;
a prime > 2 necessarily ends in
one of the digits
$1, 3, 7, 9$;
an inspection of the prime table
will show some decades
that contain all four digits

• positive integers n and $n + 2$
are called
twin primes
iff both are prime;
thus a twin prime pair is
an ordered pair $(n, n + 2)$ of prime numbers;
the sequence of twin prime pairs begins
$(3, 5), (5, 7), (11, 13), (17, 19), \cdots$;
it is conjectured but not proved (2002)
that there are infinitely many twin prime pairs

- Goldbach' s conjecture states that every even integer ≥ 4 is the sum of two primes;

eg

$4 = 2+2$

$6 = 3+3$

$8 = 3+5$

$10 = 3+7 = 5+5$

$12 = 5+7$

$14 = 3+11 = 7+7$

$16 = 3+13 = 5+11$

$18 = 5+13 = 7+11$

$20 = 3+17 = 7+13$

the conjecture has been verified up to very large even numbers but not proved (2002)

• the unique factorization theorem
states that
every positive integer n is expressible as
the product of prime numbers,
the product being unique
except for the order of the factors;
and if the factors
are written in weakly increasing order,
then the representation of n as a product of primes
is unique
and is called
the prime factorization of n

□ the first 180 primes

2	127	283	467	661	877
3	131	293	479	673	881
5	137	307	487	677	883
7	139	311	491	683	887
11	149	313	499	691	907
13	151	317	503	701	911
17	157	331	509	709	919
19	163	337	521	719	929
23	167	347	523	727	937
29	173	349	541	733	941
31	179	353	547	739	947
37	181	359	557	743	953
41	191	367	563	751	967
43	193	373	569	757	971
47	197	379	571	761	977
53	199	383	577	769	983
59	211	389	587	773	991
61	223	397	593	787	997
67	227	401	599	797	1009
71	229	409	601	809	1013
73	233	419	607	811	1019
79	239	421	613	821	1021
83	241	431	617	823	1031
89	251	433	619	827	1033
97	257	439	631	829	1039
101	263	443	641	839	1049
103	269	449	643	853	1051
107	271	457	647	857	1061
109	277	461	653	859	1063
113	281	463	659	863	1069

□ prime factorizations of the positive integers from 1 to 100

$1 = 1$

$2 = 2 \in$ prime

$3 = 3 \in$ prime

$4 = 2^2$

$5 = 5 \in$ prime

$6 = 2 \times 3$

$7 = 7 \in$ prime

$8 = 2^3$

$9 = 3^2$

$10 = 2 \times 5$

$11 = 11 \in$ prime

$12 = 2^2 \times 3$

$13 = 13 \in$ prime

$14 = 2 \times 7$

$15 = 3 \times 5$

$16 = 2^4$

$17 = 17 \in$ prime

$18 = 2 \times 3^2$

$19 = 19 \in$ prime

$20 = 2^2 \times 5$

$21 = 3 \times 7$

$22 = 2 \times 11$

$23 = 23 \in$ prime

$24 = 2^3 \times 3$

$25 = 5^2$

$26 = 2 \times 13$

$27 = 3^3$

$28 = 2^2 \times 7$

$29 = 29 \in$ prime

$30 = 2 \times 3 \times 5$

$31 = 31 \in$ prime

$32 = 2^5$

$33 = 3 \times 11$

$34 = 2 \times 17$

$35 = 5 \times 7$

$36 = 2^2 \times 3^2$

$37 = 37 \in$ prime

$38 = 2 \times 19$

$39 = 3 \times 13$

$40 = 2^3 \times 5$

$41 = 41 \in$ prime

$42 = 2 \times 3 \times 7$

$43 = 43 \in$ prime

$44 = 2^2 \times 11$

$45 = 3^2 \times 5$

$46 = 2 \times 23$

$47 = 47 \in$ prime

$48 = 2^4 \times 3$

$49 = 7^2$

$50 = 2 \times 5^2$

$51 = 3 \times 17$

$52 = 2^2 \times 13$

$53 = 53 \in$ prime

$54 = 2 \times 3^3$

$55 = 5 \times 11$

$56 = 2^3 \times 7$

$57 = 3 \times 19$

$58 = 2 \times 29$

$59 = 59 \in$ prime

$60 = 2^2 \times 3 \times 5$

$61 = 61 \in \text{prime}$

$62 = 2 \times 31$

$63 = 3^2 \times 7$

$64 = 2^6$

$65 = 5 \times 13$

$66 = 2 \times 3 \times 11$

$67 = 67 \in \text{prime}$

$68 = 2^2 \times 17$

$69 = 3 \times 23$

$70 = 2 \times 5 \times 7$

$71 = 71 \in \text{prime}$

$72 = 2^3 \times 3^2$

$73 = 73 \in \text{prime}$

$74 = 2 \times 37$

$75 = 3 \times 5^2$

$76 = 2^2 \times 19$

$77 = 7 \times 11$

$78 = 2 \times 3 \times 13$

$79 = 79 \in \text{prime}$

$80 = 2^4 \times 5$

$81 = 3^4$

$82 = 2 \times 41$

$83 = 83 \in$ prime

$84 = 2^2 \times 3 \times 7$

$85 = 5 \times 17$

$86 = 2 \times 43$

$87 = 3 \times 29$

$88 = 2^3 \times 11$

$89 = 89 \in$ prime

$90 = 2 \times 3^2 \times 5$

$91 = 7 \times 13$

$92 = 2^2 \times 23$

$93 = 3 \times 31$

$94 = 2 \times 47$

$95 = 5 \times 19$

$96 = 2^5 \times 3$

$97 = 97 \in$ prime

$98 = 2 \times 7^2$

$99 = 3^2 \times 11$

$100 = 2^2 \times 5^2$

□ counting modulo n
from the number 1 onward
in the sequence of positive integers,
every nth number is divisible by n
wh $n \in$ pos int;
this fact may be regarded as
the basis of the sieve of Eratosthenes
which automatically picks out the prime numbers
from the sequence of positive integers
by successively removing
all plural multiples of the primes

□ eight math recreational puzzle problems

- arrange the nine positive digits

into a common fraction

whose value is $\frac{1}{n}$

where n is a plural digit

&

whose numerator is the smallest possible

- solutions on the next page

- $\frac{1}{2} = \frac{6729}{13458}$

- $\frac{1}{3} = \frac{5823}{17469}$

- $\frac{1}{4} = \frac{3942}{15768}$

- $\frac{1}{5} = \frac{2697}{13485}$

- $\frac{1}{6} = \frac{2943}{17658}$

- $\frac{1}{7} = \frac{2394}{16758}$

- $\frac{1}{8} = \frac{3187}{25496}$

- $\frac{1}{9} = \frac{6381}{57429}$

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