How to Extract Square Roots the Old-Fashioned Way

#81 of Gottschalk's Gestalts

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□ we describe
the standard historical algorithm of how to
calculate/compute/determine/extract/find/get/obtain/take
the positive square roots of positive real numbers
given in decimal representation;
altho the algorithm itself is not to be considered
particularly complicated in a conceptual sense,
yet it seems to be rather recalcitrant
to being expressed verbally;
in presenting this algorithm
and seeking to understand its working,
it will be helpful to make a few general remarks
about the nature of the algorithm
before any details are offered

□ the square root sign

$\sqrt{******}$

consists of the radical sign
together with the vinculum = overbar used as aggregator
which is the only appearance of the vinculum nowadays;
the square root sign denotes the principal square root of
what is after the radical sign and under the vinculum,
this being called the ‘radicand’;
the words radix = root and vinculum = bond/tie are Latin;
they are both English loan words;
‘radical’ and ‘radicand’ are rooted in ‘radix’

□ a difficulty in describing the square root algorithm
is that
the notation of juxtaposition is used ambiguously
since
eg
juxtapostion denotes both
multiplication

ab = a×b

where a and b stand for real numbers
and
the Indo-Arabic positional numeration

ab = 10×a + b

where a and b are digits

☐ the square root algorithm consists of
an initial step followed by similar iterative steps,
each step producing one digit in the square root

each step creates a division angle;
each division angle has
• both a temporay & a permanent dividend
• both a temporary & a permanent divisor
• a quotient that is a single digit
  written above the vinculum
  & representing one place of the square root

each iterative step makes
three juxtapositions of digit blocks with
• one occurring in the dividend spot that changes
  the temporary dividend into the permanent dividend
• one occurring in the divisor spot that changes
  the temporary divisor into the permanent divisor
• one occurring above the vinculum in the square root
  & providing another digit in the square root

□ the square root algorithm
applies to a positive real number
in decimal representation
as radicand of a square root sign
and
produces the square root of the radicand
accurate to any finite number of decimal places
and
displays it over the vinculum

• initial step = step to get the iteration/recursion started
which contains five parts viz

(I1) starting with the decimal point in the radicand
tick off two-digit blocks to the left and to the right
except that the last block ticked on the left
may be only one digit

(I2) choose the greatest digit d such that
d squared is weakly less than the first block;
place d above the first block

(I3) place d squared under the first block;
subtract to get the temporary dividend

(I4) place a division angle over the temporary dividend

(I5) bring down the next block
to juxtapose the temporary dividend on its right
& to form the permanent dividend

• iterative/recursive step = step to be repeated
which contains six parts viz

(R1) double the partial square root to get the number p

(R2) place the letter r, denoting a single digit,
above the next block

(R3) juxtapose the letter r to the number p on its right
to get pr;
place pr in the divisor spot

(R4) choose the value of r as the greatest digit such that
the product of pr times r
is weakly less than the permanent dividend;
subtract this product from the permanent dividend
to get the next temporary dividend

(R5) place a division angle over the temporary dividend

(R6) bring down the next unused block
to juxtapose the temporary dividend on its right
& to form the new permanent dividend

□ we will describe in detail how to compute

$$\sqrt{1078.4656}$$

by the standard square root algorithm;
this particular radicand chosen to illustrate the algorithm is the square of a terminating decimal so that the calculation ends in a finite number of steps;
if the radicand is not a square of a terminating decimal, then the procedure is unending
and any particular finite number of decimal places in the square root may be calculated in principle

• initial step

(I1) starting with the decimal point in the radicand
tick off two-digit blocks to the left and to the right
except that the last block ticked on the left
may be only one digit

$\sqrt{10'\,78.46'\,56}$

(I2) choose the greatest digit d such that
d squared is weakly less than the first block;
place d above the first block

$$\begin{array}{l} \;\;3 \\ \sqrt{10'\,78.46'\,56} \end{array}$$

(I3) place d squared under the first block;
subtract to get the temporary dividend

$$\begin{array}{l} \;\;3 \\ \sqrt{10'\,78.46'\,56} \\ \;\;\;9 \\ \;\;\overline{\;\;1} \end{array}$$

(I4) place a division angle over the temporary dividend

$$
\begin{array}{l}
\quad 3 \\
\sqrt{10'\ 78.46'\ 56} \\
\quad\ 9 \\
\quad \overline{)1}
\end{array}
$$

(I5) bring down the next block
to juxtapose the temporary dividend on its right
& to form the permanent dividend

$$
\begin{array}{l}
\quad 3 \\
\sqrt{10'\ 78.46'\ 56} \\
\quad\ 9 \\
\quad \overline{)178}
\end{array}
$$

• first of the iterated steps

(R1) double the partial square root to get the number p

$$p = 2 \times 3 = 6$$

(R2) place the letter r, denoting a single digit,
above the next block

$$\begin{array}{l} \quad 3 \;\; r \\ \sqrt{10'\,78.46'\,56} \\ \quad 9 \\ \quad \overline{)178} \end{array}$$

(R3) juxtapose the letter r to the number p on its right to get pr;
place pr in the divisor spot

$$\begin{array}{l} \quad 3 \quad r \\ \sqrt{10'\,78.46'\,56} \\ \quad 9 \\ 6r\overline{)178} \end{array}$$

(R4) choose the value of r as the greatest digit such that the product of pr times r
is weakly less than the permanent dividend;
subtract this product from the permanent dividend
to get the next temporary dividend

$$
\begin{array}{l}
\quad 3 \quad 2 \\
\sqrt{10'\,78.46'\,56} \\
\quad\ 9 \\
62\overline{)178} \\
\quad\ 124 \\
\quad\ \overline{\ \ 54}
\end{array}
$$

(R5) place a division angle over the temporary dividend

3   2

$\sqrt{10'\ 78.46'\ 56}$

9

$62\overline{)178}$

124

$\overline{)\ 54}$

(R6) bring down the next unused block
to juxtapose the temporary dividend on its right
& to form the new permanent dividend

$$
\begin{array}{l}
\quad 3 \quad 2 \\
\sqrt{10'\,78.46'\,56} \\
\quad 9 \\
62\overline{)178} \\
\quad 124 \\
\quad \overline{)\ 5446}
\end{array}
$$

• second of the iterated steps

(R1) double the partial square root to get the number p

$$p \;=\; 2 \times 32 \;=\; 64$$

(R2) place the letter r, denoting a single digit,
above the next block

$$\begin{array}{l}
\quad\;\; 3 \;\; 2 \;\; r \\
\;\;\sqrt{10'\,78.46'\,56} \\
\qquad 9 \\
62\overline{)178} \\
\qquad 124 \\
\quad\;\; \overline{)\;5446}
\end{array}$$

(R3) juxtapose the letter r to the number p on its right to get pr;
place pr in the divisor spot

$$
\begin{array}{l}
\quad 3 \quad 2 \quad r \\
\sqrt{10'\,78.46'\,56} \\
\quad 9 \\
62\overline{)178} \\
\quad 124 \\
64r\overline{)\;5446}
\end{array}
$$

(R4) choose the value of r as the greatest digit such that
the product of pr times r
is weakly less than the permanent dividend;
subtract this product from the permanent dividend
to get the next temporary dividend

$$
\begin{array}{l}
\quad 3 \quad 2 \quad 8 \\
\sqrt{10'\,78.46'\,56} \\
\quad 9 \\
62\overline{)178} \\
\quad\;\; 124 \\
648\overline{)\;5446} \\
\qquad\;\; 5184 \\
\qquad\;\; \overline{\;\;\;262}
\end{array}
$$

(R5) place a division angle over the temporary dividend

$$
\begin{array}{l}
\quad\;\; 3 \quad 2 \quad 8 \\
\sqrt{10'\,78.46'\,56} \\
\quad\;\; 9 \\
62\overline{)178} \\
\qquad 124 \\
648\overline{)\;5446} \\
\qquad\;\; 5184 \\
\qquad \overline{)\;\;\;262}
\end{array}
$$

(R6) bring down the next unused block
to juxtapose the temporary dividend on its right
& to form the new permanent dividend

$$
\begin{array}{rl}
& \;\;3\;\;\;2\;\;\;8 \\
& \sqrt{10'\,78.46'\,56} \\
& \;\;9 \\
62) & 178 \\
& 124 \\
648) & \;5446 \\
& \;\;5184 \\
) & \;\;\;26256
\end{array}
$$

• third of the iterated steps

(R1) double the partial square root to get the number p

$$p = 2 \times 328 = 656$$

(R2) place the letter r, denoting a single digit,
above the next block

$$\begin{array}{rl}
 & \;\;3\;\;\;2\;\;\;8\;\;\;r \\
 & \sqrt{10'\,78.46'\,56} \\
 & \;\;9 \\
62 & \overline{)178} \\
 & \;\;124 \\
648 & \overline{)\;5446} \\
 & \;\;\;5184 \\
 & \overline{)\;\;26256}
\end{array}$$

(R3) juxtapose the letter r to the number p on its right
to get pr;
place pr in the divisor spot

3 2 8 r

$\sqrt{10'\,78.46'\,56}$

9

$62\overline{)178}$

124

$648\overline{)\ 5446}$

5184

$656r\overline{)\ \ 26256}$

(R4) choose the value of r as the greatest digit such that
the product of pr times r
is weakly less than the permanent dividend;
subtract this product from the permanent dividend
to get the next temporary dividend which is zero

the procedure ends
meaning that any further iterations will produce
only zeros in the square root
&
an exact square root is obtained

$$
\begin{array}{rl}
 & \;\;3\;\;\;2\;\;\;8\;\;\;4 \\
 & \sqrt{10'\,78.46'\,56} \\
 & \;\;9 \\
62) & \overline{178} \\
 & 124 \\
648) & \overline{5446} \\
 & 5184 \\
6564) & \overline{26256} \\
 & \underline{26256} \\
 & 0
\end{array}
$$

$\therefore \sqrt{1078.4656} = 32.84$