Basic Blocks
In Equality & Inequality & Diagram

#57 of Gottschalk's Gestalts

A Series Illustrating Innovative Forms
of the Organization & Exposition
of Mathematics
by Walter Gottschalk

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GG57-2

□ some notable
canonical unit blocks
in the number line $\mathbb{R}$
&
in the number plane $\mathbb{R} \times \mathbb{R} = \mathbb{C}$

• the closed unit interval
= the unit interval
$=_{ab}$ cl un int
$=_{ab}$ un int
$=_{df} \{x \mid x \in \mathbb{R} \ \& \ 0 \le x \le 1\}$
$= \mathbb{R}[0,1]$
$=_{dn} \mathbb{I}$
$=_{rd}$ (open cap) eye
wh
$\mathbb{I} \leftarrow$ initial letter of ' interval'

• the open unit interval

$=_{ab}$ op un int

$=_{df} \{x \mid x \in \mathbb{R} \ \& \ 0 < x < 1\}$

$= \mathbb{R}(0,1)$

$=_{dn} \overset{o}{\mathbb{I}}$

$=_{rd}$ (open cap) eye (overscript) oh

wh

$\overset{o}{\mathbb{I}} \leftarrow \mathbb{I}$ & overscript interior operator $\circ$

• the closed unit square in quadrant I

= the unit square in quadrant I

$=_{ab}$ cl un sq in QI

$=_{ab}$ un sq in QI

$=_{df} \{(x, y) \mid x, y \in \mathbb{R} \ \& \ 0 \leq x \leq 1 \ \& \ 0 \leq y \leq 1\}$

$= \{z \mid z \in \mathbb{C} \ \& \ 0 \leq \Re z \leq 1 \ \& \ 0 \leq \Im z \leq 1\}$

$= \mathbb{I} \times \mathbb{I}$

$= \mathbb{I}^2$

• the open unit square in quadrant I

$=_{ab}$ op un sq in QI

$=_{df} \{(x,y) | x,y \in \mathbb{R} \ \& \ 0 < x < 1 \ \& \ 0 < y < 1\}$

$= \{z | z \in \mathbb{C} \ \& \ 0 < \Re z < 1 \ \& \ 0 < \Im z < 1\}$

$= \overset{o}{\mathbb{C}} \times \overset{o}{\mathbb{C}}$

$= \overset{o}{\mathbb{C}}{}^2$

- the unit square frame in quadrant I

$=_{ab}$ un sq fr in QI

$$=_{df} \{ (x,y) | x,y \in \mathbb{R} \ \& \ 0 \le x \le 1 \ \& \ 0 \le y \le 1$$
$$\& \ (x = 0 \text{ or } x = 1 \text{ or } y = 0 \text{ or } y = 1) \}$$

$$= \{ z | z \in \mathbb{C} \ \& \ 0 \le \Re z \le 1 \ \& \ 0 \le \Im z \le 1$$
$$\& \ (\Re z = 0 \text{ or } \Re z = 1 \text{ or } \Im z = 0 \text{ or } \Im z = 1) \}$$

- the closed unit diamond

= the unit diamond

$=_{ab}$ un dia

$=_{df} \{(x,y) \mid x,y \in \mathbb{R} \ \& \ |x| + |y| \leq 1\}$

$= \{z \mid z \in \mathbb{C} \ \& |\Re z| + |\Im z| \leq 1\}$

• the open unit diamond

$=_{ab}$ op un dia

$=_{df} \left\{(x,y) \mid x,y \in \mathbb{R} \ \& \ |x| + |y| < 1\right\}$

$= \left\{z \mid z \in \mathbb{C} \ \& \ |\Re z| + |\Im z| < 1\right\}$

• the unit diamond frame

$=_{ab}$ un dia fr

$=_{df} \{(x,y) \mid x,y \in \mathbb{R} \ \& \ |x| + |y| = 1\}$

$= \{z \mid z \in \mathbb{C} \ \& \ |\Re z| + |\Im z| = 1\}$

• the closed unit corner in quadrant I

= the unit corner in quadrant I

$=_{ab}$ cl un cr in QI

$=_{ab}$ un cr in QI

$=_{df} \{(x,y) \mid x,y \in \mathbb{R} \ \& \ x \geq 0 \ \& \ y \geq 0 \ \& \ x+y \leq 1\}$

$= \{z \mid z \in \mathbb{C} \ \& \ \Re z \geq 0 \ \& \ \Im z \geq 0 \ \& \ \Re z + \Im z \leq 1\}$

- the open unit corner in quadrant I

$=_{ab}$ op un cr in QI

$=_{df} \{(x,y) | x,y \in \mathbb{R} \ \& \ x > 0 \ \& \ y > 0 \ \& \ x + y < 1\}$

$= \{z | z \in \mathbb{C} \ \& \ \Re z > 0 \ \& \ \Im z > 0 \ \& \ \Re z + \Im z < 1\}$

- the unit corner frame in quadrant I

$=_{ab}$ un cr fr in QI

$=_{df} \{ (x,y) \mid x,y \in \mathbb{R} \ \& \ 0 \le x \le 1 \ \& \ 0 \le y \le 1$

$\qquad \& \ (x = 0 \text{ or } y = 0 \text{ or } x + y = 1) \}$

$= \{ z \mid z \in \mathbb{C} \ \& \ 0 \le \Re z \le 1 \ \& \ 0 \le \Im z \le 1$

$\qquad \& \ (\Re z = 0 \text{ or } \Im z = 0 \text{ or } \Re z + \Im z = 1) \}$

- the closed unit disk

= the unit disk

$=_{ab}$ cl un ds

$=_{ab}$ un ds

$=_{df} \left\{(x, y) \mid x, y \in \mathbb{R} \;\&\; x^2 + y^2 \leq 1\right\}$

$= \{z \mid z \in$ 👍 $\& \; |z| \leq 1\}$

$=_{dn} \mathbb{D}$

$=_{rd}$ (open cap) dee

wh

$\mathbb{D} \leftarrow$ initial letter of 'disk'

• the open unit disk

$=_{ab}$ op un ds

$=_{df} \left\{(x,y) \mid x,y \in \mathbb{R} \ \& \ x^2 + y^2 < 1\right\}$

$= \left\{z \mid z \in \mathbb{C} \ \& \ |z| < 1\right\}$

$=_{dn} \overset{o}{\mathbb{D}}$

$=_{rd}$ (open cap) dee (overscript) oh

wh

$\overset{o}{\mathbb{D}} \leftarrow \mathbb{D}$ & overscript interior operator $\circ$

• the unit circle

$=_{ab}$ un cir

$=_{df} \{(x,y) \mid x,y \in \mathbb{R} \ \& \ x^2 + y^2 = 1\}$

$= \{z \mid z \in \mathbb{C} \ \& \ |z| = 1\}$

$=_{dn} \mathbb{T}$

$=_{rd}$ (open cap) tee

wh

$\mathbb{T} \leftarrow$ initial letter of 'torus'

because

$\mathbb{T}^n$ denotes the n - torus (n $\in$ pos int)

&

the circle is the 1 - torus

- the right closed unit semidisk

= the right unit semidisk

$=_{ab}$ rt cl un sd

$=_{ab}$ rt un sd

$$=_{df} \left\{(x,y) \;\middle|\; x,y \in \mathbb{R} \;\&\; x \geq 0 \;\&\; x^2 + y^2 \leq 1\right\}$$

$$= \left\{z \;\middle|\; z \in \mathbb{C} \;\&\; \Re z \geq 0 \;\&\; |z| \leq 1\right\}$$

- the right open unit semidisk

$=_{ab}$ rt op un sd

$=_{df} \left\{ (x,y) \,\middle|\, x,y \in \mathbb{R} \ \& \ x > 0 \ \& \ x^2 + y^2 < 1 \right\}$

$= \left\{ z \,\middle|\, z \in \mathbb{C} \ \& \ \Re z > 0 \ \& \ |z| < 1 \right\}$

- the right closed unit semicircle

= the right unit semicircle

$=_{ab}$ rt cl un sc

$=_{ab}$ rt un sc

$=_{df}$ $\left\{(x,y) \;\middle|\; x,y \in \mathbb{R} \;\&\; x \geq 0 \;\&\; x^2 + y^2 = 1\right\}$

$=$ $\left\{z \;\middle|\; z \in \mathbb{C} \;\&\; \Re z \geq 0 \;\&\; |z| = 1\right\}$

- the right open unit semicircle

$=_{ab}$ rt op un sc

$=_{df} \left\{(x,y) \,\middle|\, x,y \in \mathbb{R} \ \& \ x > 0 \ \& \ x^2 + y^2 = 1\right\}$

$= \left\{z \mid z \in \mathbb{C} \ \& \ \Re z > 0 \ \& \ |z| = 1\right\}$

- the left closed unit semidisk

= the left unit semidisk

$=_{ab}$ lt cl un sd

$=_{ab}$ lt un sd

$=_{df}$ $\left\{(x,y) \,\middle|\, x,y \in \mathbb{R} \ \&\ x \le 0 \ \&\ x^2 + y^2 \le 1\right\}$

= $\left\{z \,\middle|\, z \in \mathbb{C} \ \&\ \Re z \le 0 \ \&\ |z| \le 1\right\}$

- the left open unit semidisk

$=_{ab}$ lt op un sd

$=_{df} \left\{(x,y) \,\middle|\, x,y \in \mathbb{R} \ \& \ x < 0 \ \& \ x^2 + y^2 < 1 \right\}$

$= \left\{z \,\middle|\, z \in \mathbb{C} \ \& \ \Re z < 0 \ \& \ |z| < 1 \right\}$

• the left closed unit semicircle

= the left unit semicircle

$=_{ab}$ lt cl un sc

$=_{ab}$ lt un sc

$=_{df}$ $\left\{(x,y) \mid x,y \in \mathbb{R} \ \& \ x \leq 0 \ \& \ x^2 + y^2 = 1\right\}$

$= \left\{z \mid z \in \mathbb{C} \ \& \ \Re z \leq 0 \ \& \ |z| = 1\right\}$

- the left open unit semicircle

$=_{ab}$ lt op un sc

$=_{df} \left\{(x,y) \,\middle|\, x,y \in \mathbb{R} \;\&\; x < 0 \;\&\; x^2 + y^2 = 1 \right\}$

$= \left\{z \mid z \in \mathbb{C} \;\&\; \Re z < 0 \;\&\; |z| = 1\right\}$

• the upper closed unit semidisk

= the upper unit semidisk

$=_{ab}$ up cl un sd

$=_{ab}$ up un sd

$=_{df}$ $\left\{(x,y) \;\middle|\; x,y \in \mathbb{R} \;\&\; y \geq 0 \;\&\; x^2 + y^2 \leq 1\right\}$

$= \left\{z \;\middle|\; z \in \mathbb{C} \;\&\; \Im z \geq 0 \;\&\; |z| \leq 1\right\}$

• the upper open unit semidisk

$=_{ab}$ up op un sd

$=_{df} \left\{ (x,y) \,\middle|\, x,y \in \mathbb{R} \;\&\; y > 0 \;\&\; x^2 + y^2 < 1 \right\}$

$= \left\{ z \mid z \in \mathbb{C} \;\&\; \Im z > 0 \;\&\; |z| < 1 \right\}$

- the upper closed unit semicircle

= the upper unit semicircle

$=_{ab}$ up cl un sc

$=_{ab}$ up un sc

$=_{df}$ $\left\{(x,y) \;\middle|\; x,y \in \mathbb{R} \;\&\; y \geq 0 \;\&\; x^2 + y^2 = 1\right\}$

$= \left\{z \mid z \in \mathbb{C} \;\&\; \Im z \geq 0 \;\&\; |z| = 1\right\}$

- the upper open unit semicircle

$=_{ab}$ up op un sc

$=_{df}$ $\left\{(x,y)\;\middle|\; x,y \in \mathbb{R} \;\&\; y > 0 \;\&\; x^2 + y^2 = 1\right\}$

$= \left\{z \mid z \in \mathbb{C} \;\&\; \Im z > 0 \;\&\; |z| = 1\right\}$

• the lower closed unit semidisk

= the lower unit semidisk

$=_{ab}$ lo cl un sd

$=_{ab}$ lo un sd

$=_{df}$ $\left\{(x,y) \,\middle|\, x,y \in \mathbb{R} \;\&\; y \le 0 \;\&\; x^2 + y^2 \le 1\right\}$

= $\left\{z \,\middle|\, z \in \mathbb{C} \;\&\; \Im z \le 0 \;\&\; |z| \le 1\right\}$

- the lower open unit semidisk

$=_{ab}$ lo op un sd

$=_{df} \left\{(x,y) \,\middle|\, x,y \in \mathbb{R} \;\&\; y < 0 \;\&\; x^2 + y^2 \le 1\right\}$

$= \left\{z \,\middle|\, z \in \mathbb{C} \;\&\; \Im z < 0 \;\&\; |z| \le 1\right\}$

• the lower closed unit semicircle

= the lower unit semicircle

$=_{ab}$ lo cl un sc

$=_{ab}$ lo un sc

$=_{df}$ $\left\{(x,y) \,\middle|\, x,y \in \mathbb{R} \;\&\; y \leq 0 \;\&\; x^2 + y^2 = 1\right\}$

$= \left\{z \,\middle|\, z \in \mathbb{C} \;\&\; \Im z \leq 0 \;\&\; |z| = 1\right\}$

- the lower open unit semicircle

$=_{ab}$ lo op un sc

$=_{df} \left\{(x,y) \,\middle|\, x,y \in \mathbb{R} \;\&\; y < 0 \;\&\; x^2 + y^2 = 1\right\}$

$= \left\{z \,\middle|\, z \in \mathbb{C} \;\&\; \Im z < 0 \;\&\; |z| = 1\right\}$

- the closed unit quarterdisk in quadrant I

= the unit quarterdisk in quadrant I

$=_{ab}$ cl un qd in QI

$=_{ab}$ un qd in QI

$=_{df}$ $\left\{(x, y) \mid x, y \in \mathbb{R} \ \& \ x \geq 0 \ \& \ y \geq 0 \ \& \ x^2 + y^2 \leq 1\right\}$

$= \left\{z \mid z \in \mathbb{C} \ \& \ \Re z \geq 0 \ \& \ \Im z \geq 0 \ \& \ |z| \leq 1\right\}$

- the open unit quarterdisk in quadrant I

$=_{ab}$ op un qd in QI

$=_{df} \left\{(x,y) \mid x,y \in \mathbb{R} \ \& \ x > 0 \ \& \ y > 0 \ \& \ x^2 + y^2 < 1\right\}$

$= \left\{z \mid z \in \mathbb{C} \ \& \ \Re z > 0 \ \& \ \Im z > 0 \ \& \ |z| < 1\right\}$

• the closed unit quartercircle in quadrant I

= the unit quartercircle in quadrant I

$=_{ab}$ cl un qc in QI

$=_{ab}$ un qc in QI

$=_{df}$ $\{(x,y) \mid x,y \in \mathbb{R} \ \& \ x \geq 0 \ \& \ y \geq 0 \ \& \ x^2 + y^2 = 1\}$

$= \{z \mid z \in \mathbb{C} \ \& \ \Re z \geq 0 \ \& \ \Im z \geq 0 \ \& \ |z| = 1\}$

• the open unit quartercircle in quadrant I

$=_{ab}$ op un qc in QI

$=_{df}$ $\left\{(x, y) \mid x, y \in \mathbb{R} \ \& \ x > 0 \ \& \ y > 0 \ \& \ x^2 + y^2 = 1\right\}$

$= \left\{z \mid z \in \mathbb{C} \ \& \ \Re z > 0 \ \& \ \Im z > 0 \ \& \ |z| = 1\right\}$

• there are similar definitions & diagrams
for squares, corners, quarterdisks, quartercircles
in quadrants II, III, IV

• let n be an integer ≥ 3
&
let z be a complex variable;
then
the n complex roots
of the nth degree cyclotomic equation
$z^n = 1$
which are
the n distinct nth complex roots of unity
constitute the n vertices
of a regular n - sided polygon
with center at the origin
and
with one vertex at 1;
thus there are automatically defined:
the closed unit n - gon,
the open unit n - gon,
the unit n - gon frame;
the circumcircle of this polygon
is the unit circle

• etymology

cyclotomy / cyclotomic

↑

κυκλος (Greek) = circle, ring, wheel

+

τομη (Greek) = cutting, section