The Geometry of Compass Directions

#54 of Gottschalk's Gestalts

A Series Illustrating Innovative Forms
of the Organization & Exposition
of Mathematics
by Walter Gottschalk

GG54-1 (28)

GG54-2

□ the geometry of plane directions
based on the mariner's compass

- this is a unit of angle / arc

  that is primarily of nautical use:

  the point as angle

= the point angle

= the point

= one thirty - seconds of a round angle

  measured in degrees

$$= \frac{1}{32} \times 360^\circ$$

= eleven and one - quarter degrees

$= 11^\circ \, 15'$

whence

one point	=	$11^\circ \, 15'$
two points	=	$22^\circ \, 30'$
four points	=	45°
eight points	=	90°
sixteen points	=	180°
thirty – two points	=	360°

• as a universal notational device
the (often capitalized) initial letter,
and possibly other letters,
of the name of an object
may serve as
the denotation of the object

• the compass card
= dn CC
= df the set of all directions in the plane

• think of a direction in the plane
as consisting of a maximal set
of similarly directed parallel lines;
thus a single directed line in the plane
uniquely determines
a direction in the plane
viz
the set of all directed lines
strictly parallel to
the given directed line

• a direction in the plane may be represented by
a ray from an arbitrarily chosen fixed point of the plane
which is called
the origin O,
where O
is the capitalized initial letter
of the word origin

GG54-4

• the compass circle
= dn C (from the initial letters)
= df the circle
with center at O
and
with some fixed radius say unity

• there are evident canonical correspondences among:
directions in the plane,
rays from the origin O,
radii of the compass circle C,
points of the compass circle C

• a point of the compass
= a compass point
= a point
= df a point of the compass circle C
which by the above canonical correspondences
may be equivalently regarded as
a direction in the plane
or
a ray from the origin
or
a radius of the compass circle C

• the four cardinal compass points
with denotation
by the capitalized initial letters
are defined to be:
north = N
east = E
south = S
west = W
in the clockwise rotary direction
and
a quadrant = a right angle = eight points
apart consecutively,
the north point being chosen first say
and once chosen remaining fixed thruout the discussion

• note that
the words north, east, south , west
are also descriptive of directions
in the compass circle
toward the points N, E, S, W

• GP

N

W   E

S
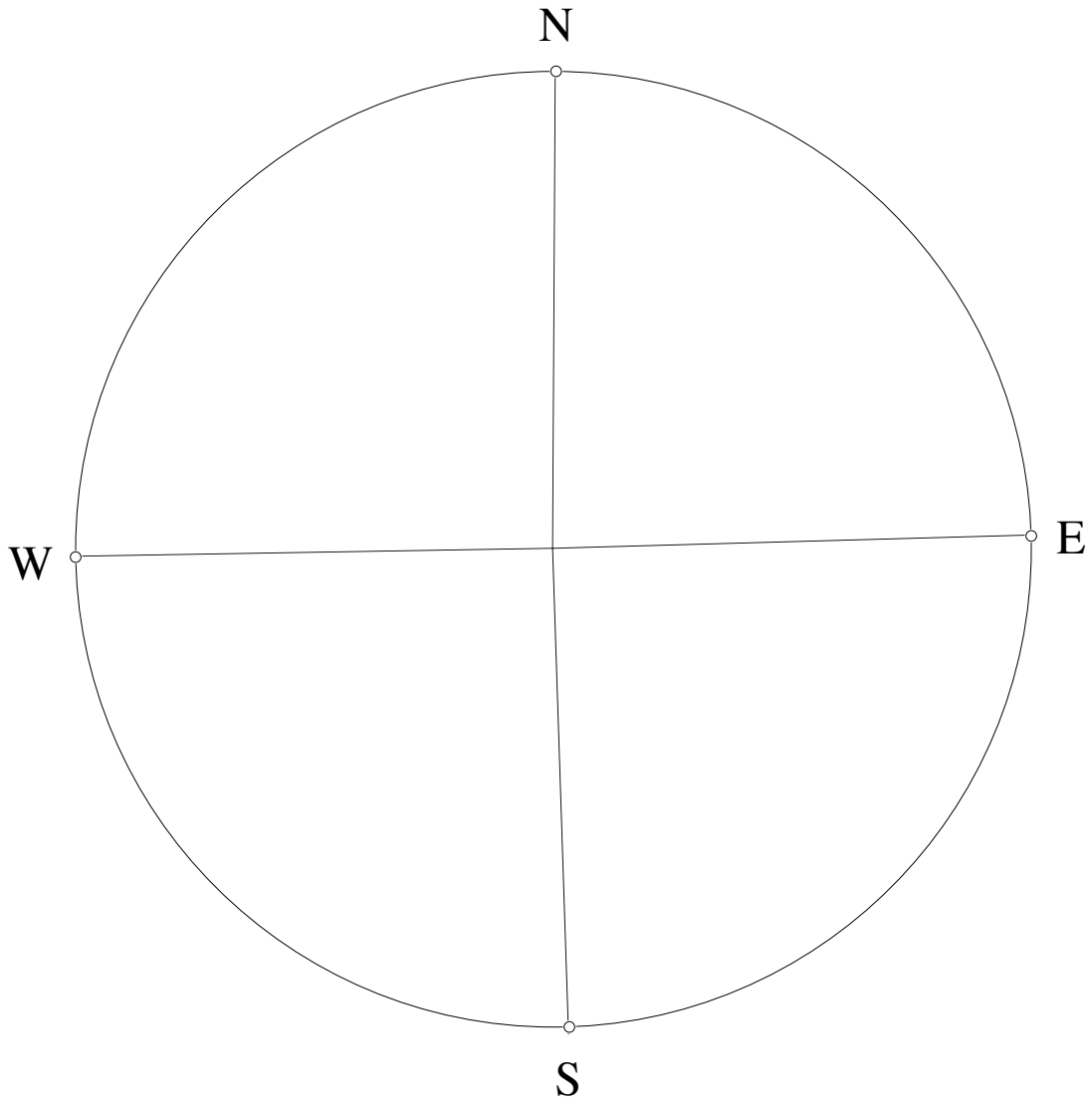
the four cardinal compass points

GG54-7

• the four first-order intercardinal compass points are defined and denoted as follows:

northeast
= NE
= the directed bisector of the right angle NOE
= the midpoint of the quarter compass circle arc
   from point N to point E
= halfway from N to E
= four points east of north

northwest
= NW
= the directed bisector of the right angle NOW
= the midpoint of the quarter compass circle arc
   from point N to point W
= halfway from N to W
= four points west of north

southeast
= SE
= the directed bisector of the right angle SOE
= the midpoint of the quarter compass circle arc
   from point S to point E
= halfway from S to E
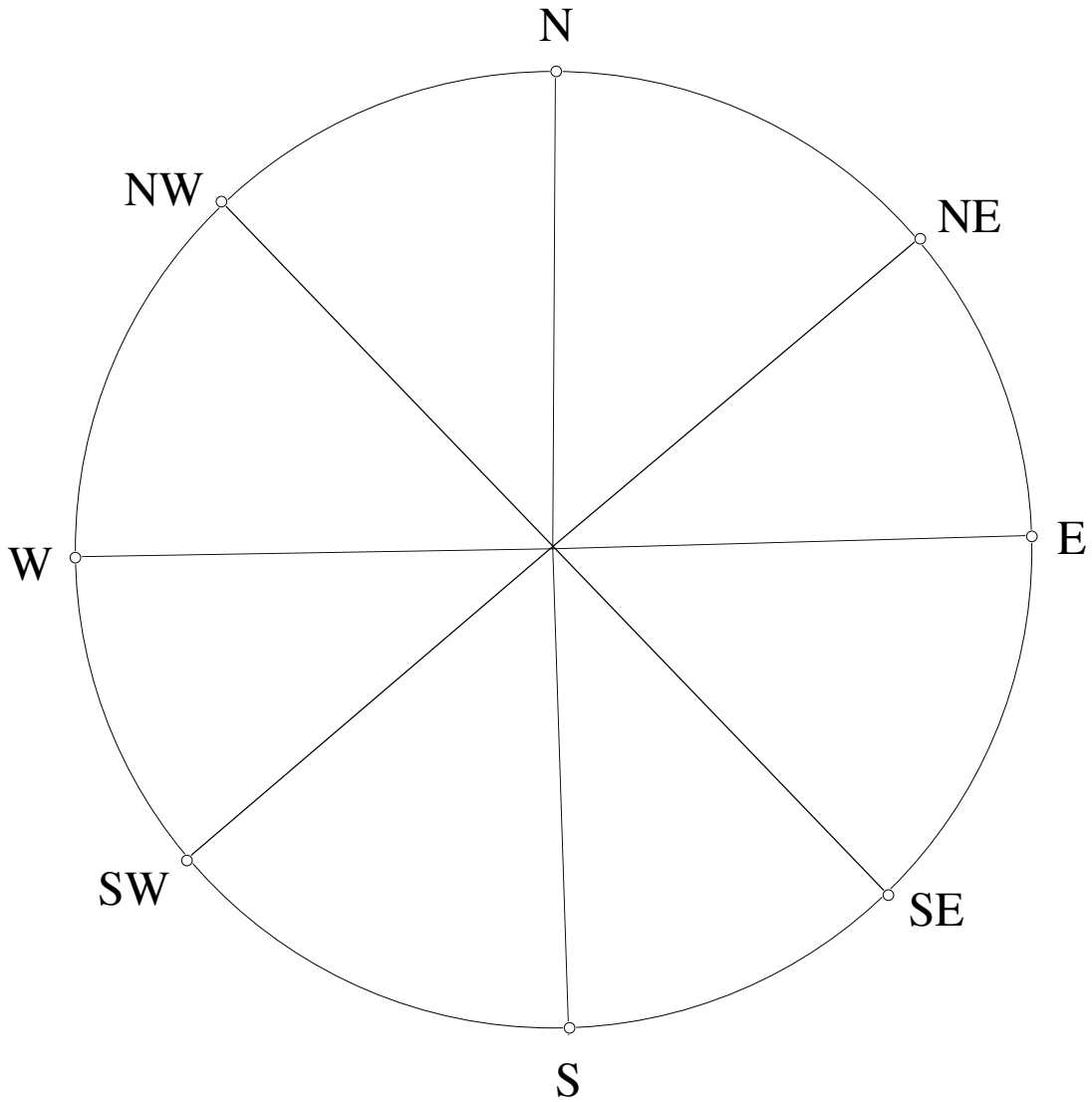= four points east of south

southwest
= SW
= the directed bisector of the right angle SOW
= the midpoint of the quarter compass circle arc
   from point S to point W
= halfway from S to W
= four points west of south

• GP

N

NW

NE

W

E

SW

SE

S

the four cardinal compass points
&
the four first-order intercardinal compass points

GG54-10

• the eight second-order intercardinal compass points are defined and denoted as follows:

north-northeast
= NNE
= the midpoint of the 45º compass circle arc
  from point N to point NE
= halfway from N to NE
= two points east of north

north-northwest
= NNW
= the midpoint of the 45º compass circle arc
  from point N to point NW
= halfway from N to NW
= two points west of north

south-southeast
= SSE
= the midpoint of the 45º compass circle arc
  from point S to point SE
= halfway from S to SE
= two points east of south

south-southwest
= SSW
= the midpoint of the 45º compass circle arc
  from point S to point SW
= halfway from S to SW
= two points west of south

GG54-11

east-northeast
= ENE
= the midpoint of the 45º compass circle arc
   from point E to point NE
= halfway from E to NE
= two points north of east

east-southeast
= ESE
= the midpoint of the 45º compass circle arc
   from point E to point SE
= halfway from E to SE
= two points south of east

west-northwest
= WNW
= the midpoint of the 45º compass circle arc
   from point W to point NW
= halfway from W to NW
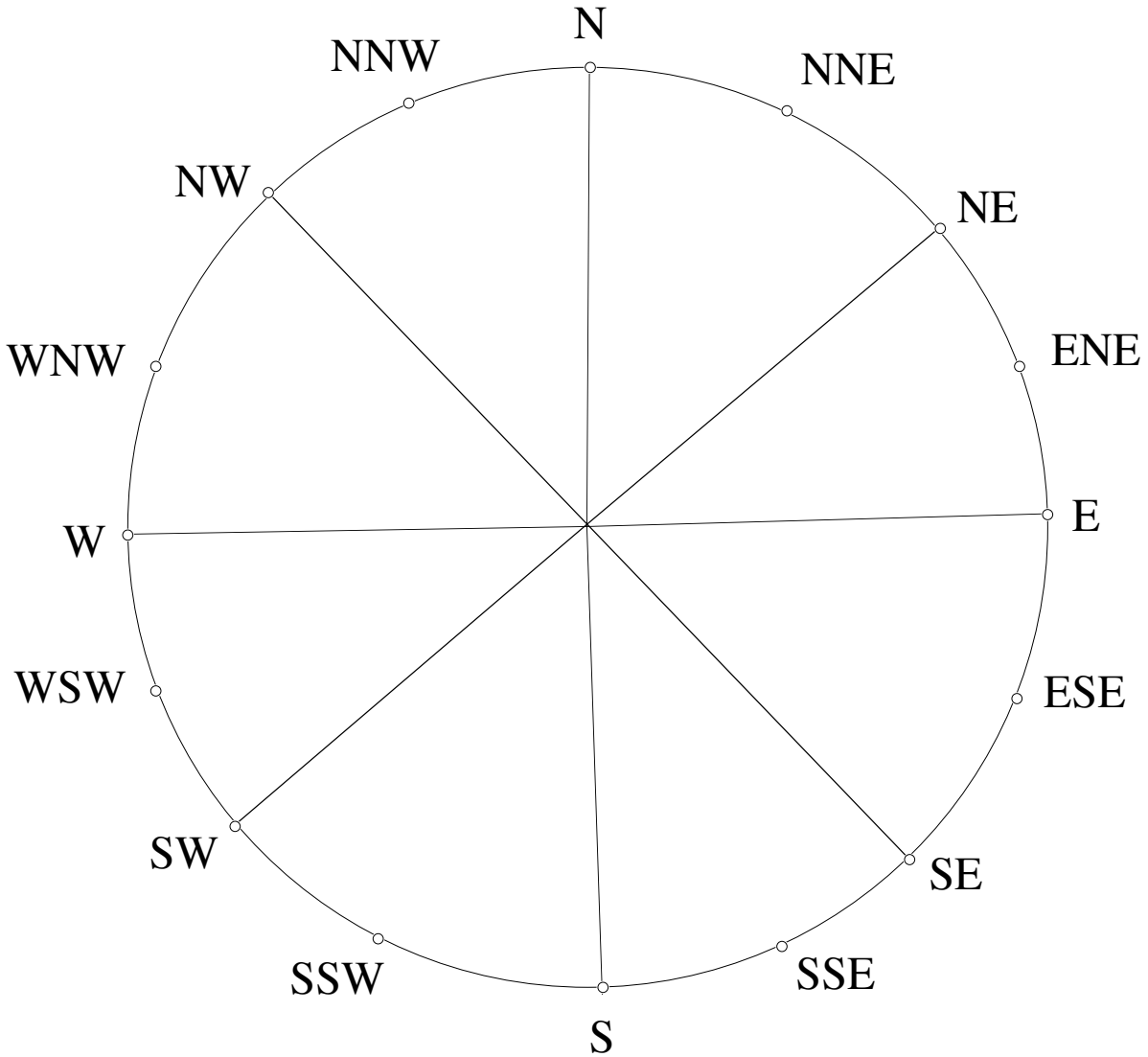= two points north of west

west-southwest
= WSW
= the midpoint of the 45º compass circle arc
   from point W to point SW
= halfway from W to SW
= two points south of west

GG54-12

• GP

the four cardinal compass points
&
the four first-order intercardinal compass points
&
the eight second-order intercardinal compass points

GG54-13

• the sixteen third-order intercardinal compass points are defined and denoted as follows:

north by east
= NbE
= the midpoint of the 22º 30′ compass circle arc
  from point N to point NNE
= halfway from N to NNE
= one point east of north

north by west
= NbW
= the midpoint of the 22º 30′ compass circle arc
  from point N to point NNW
= halfway from N to NNW
= one point west of north

south by east
= SbE
= the midpoint of the 22º 30′ compass circle arc
  from point S to point SSE
= halfway from S to SSE
= one point east of south

south by west
= SbW
= the midpoint of the 22º 30′ compass circle arc
  from point S to point SSW
= halfway from S to SSW
= one point west of south   GG54-14

east by north
= EbN

= the midpoint of the 22º 30′ compass circle arc
   from point E to point ENE
= halfway from E to ENE
= one point north of east

east by south
= EbS

= the midpoint of the 22º 30′ compass circle arc
   from point E to point ESE
= halfway from E to ESE
= one point south of east

west by north
= WbN

= the midpoint of the 22º 30′ compass circle arc
   from point W to point WNW
= halfway from W to WNW
= one point north of west

west by south
= WbS

= the midpoint of the 22º 30′ compass circle arc
   from point W to point WSW
= halfway from W to WSW
= one point south of west

GG54-15

northeast by north

= NEbN

= the midpoint of the 22º 30′ compass circle arc
  from point NE to point NNE
= halfway from NE to NNE
= one point north of northeast

northeast by east

= NEbE

= the midpoint of the 22º 30′ compass circle arc
  from point NE to point ENE
= halfway from NE to ENE
= one point east of northeast

northwest by north

= NWbN

= the midpoint of the 22º 30′ compass circle arc
  from point NW to point NNW
= halfway from NW to NNW
= one point north of northwest

northwest by west

= NWbW

= the midpoint of the 22º 30′ compass circle arc
  from point NW to point WNW
= halfway from NW to WNW
= one point west of northwest

southeast by south
= SEbS
= the midpoint of the 22º 30′ compass circle arc
  from point SE to point SSE
= halfway from SE to SSE
= one point south of southeast

southeast by east
= SEbE
= the midpoint of the 22º 30′ compass circle arc
  from point SE to point ESE
= halfway from SE to ESE
= one point east of southeast

southwest by south
= SWbS
= the midpoint of the 22º 30′ compass circle arc
  from point SW to point SSW
= halfway from SW to SSW
= one point south of southwest

southwest by west
= SWbW
= the midpoint of the 22º 30′ compass circle arc
  from point SW to point WSW
= halfway from SW to WSW
= one point west of southwest

• the 32 compass points defined above
viz

the four cardinal points:
N, S. E, W

the four first-order intercardinal points:
NE, NW, SE, SW

the eight second-order intercardinal points:
NNE, NNW, SSE, SSW,
ENE, ESE, WNW, WSW

the sixteen third-order intercardinal points:
NbE, NbW, SbE, SbW,
EbN, EbS, WbN, WbS,
NEbN, NEbE, NWbN, NWbW,
SEbS, SEbE, SWbS, SWbW

may all be called the rose points
and together
their set constitutes
the compass rose

• the azimuth of a direction
is the sectorial angle
measured from north
clockwise
to the given direction

• the quadrantal notation for a direction
is the properly lettered acute angle
measured from north or south
toward east or west
to the given direction

• below are listed the 32 rose points
together with
their azimuth in points & degrees
and
their quadrantral notation;
the listing is in increasing order of azimuth;
such a cyclic listing
of the 32 consecutive compass/rose points
is called
'boxing the compass'

• boxing the compass

N $= 0\text{ p} = 0^0$ $= \text{N}$

NbE $= 1\text{ p} = 11^0\,15'$ $= \text{N}\,11^0\,15'\text{ E}$

NNE $= 2\text{ p} = 22^0\,30'$ $= \text{N}\,22^0\,30'\text{ E}$

NEbN $= 3\text{ p} = 33^0\,45'$ $= \text{N}\,33^0\,45'\text{ E}$

NE $= 4\text{ p} = 45^o$ $= \text{N}\,45^o\text{ E}$

NEbE $= 5\text{ p} = 56^o\,15'$ $= \text{N}\,56^o\,15'\text{ E}$

ENE $= 6\text{ p} = 67^o\,30'$ $= \text{N}\,67^o\,30'\text{ E}$

EbN $= 7\text{ p} = 78^o\,45'$ $= \text{N}\,78^o\,45'\text{ E}$

name	symbol
north	N
north by east	NbE
north-northeast	NNE
northeast by north	NEbN
northeast	NE
northeast by east	NEbE
east-northeast	ENE
east by north	EbN

GG54-21

E $= 8\,p = 90^0$ $= E$

EbS $= 9\,p = 101^0\,15'$ $= S\,78^0\,45'\,E$

ESE $= 10\,p = 112^0\,30'$ $= S\,67^0\,30'\,E$

SEbE $= 11\,p = 123^0\,45'$ $= S\,56^0\,15'\,E$

SE $= 12\,p = 135^o$ $= S\,45^o\,E$

SEbS $= 13\,p = 146^o\,15'$ $= S\,33^o\,45'\,E$

SSE $= 14\,p = 157^o\,30'$ $= S\,22^o\,30'\,E$

SbE $= 15\,p = 168^o\,45'$ $= S\,11^o\,15'\,E$

GG54-22

name	symbol
east	E
east by south	EbS
east-southeast	ESE
southeast by east	SEbE
southeast	SE
southeast by south	SEbS
south-southeast	SSE
south by east	SbE

$$S \qquad = 16\,p = 180^0 \qquad = S$$

$$SbW \quad = 17\,p = 191^0\,15' \;\; = S\,11^0\,15'\,W$$

$$SSW \quad = 18\,p = 202^0\,30' \;\; = S\,22^0\,30'\,W$$

$$SWbS \; = 19\,p = 213^0\,45' \;\; = S\,33^0\,45'\,W$$

$$SW \qquad = 20\,p = 225^o \qquad = S\,45^o\,W$$

$$SWbW = 21\,p \;\; = 236^o\,15' \;\; = S\,56^o\,15'\,W$$

$$WSW \quad = 22\,p = 247^o\,30' \;\; = S\,67^o\,30'\,W$$

$$WbS \quad = 23\,p = 258^o\,45' \;\; = S\,78^o\,45'\,W$$

GG54-24

name	symbol
south	S
south by west	SbW
south-southwest	SSW
southwest by south	SWbS
southwest	SW
southwest by west	SWbW
west-southwest	WSW
west by south	WbS

GG54-25

W   = 24 p = 270^0 = W

WbN   = 25 p = $281^0\ 15'$ = $N78^0\ 45'\ W$

WNW   = 26 p = $292^0\ 30'$ = $N\,67^0\ 30'\ W$

NWbW = 27 p = $303^0\ 45'$ = $N\,56^0\ 15'\ W$

NW   = 28 p = 315° = $N\,45^o\ W$

NWbN = 29 p = $326^o\ 15'$ = $N\,33^o\ 45'\ W$

NNW   = 30 p = $337^o\ 30'$ = $N\,22^o\ 30'\ W$

NbW   = 31 p = $348^o\ 45'$ = $N\,11^o\ 15'\ W$

GG54-26

name	symbol
west	W
west by north	WbN
west-northwest	WNW
northwest by west	NWbW
northwest	NW
northwest by north	NWbN
north-westwest	NNW
north by west	NbW

• the first sixteen words for compass directions
as defined above
can serve as bases for additional words;
here are examples for the word north:
north-bound
northerly
northern
Northerner
northing
northland
northward
northwards