Alphabetic Runs Make Good Signs

#50 of Gottschalk's Gestalts

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☐ alphabetic runs make good signs

a consecutive alphabetic run
of three or more lowercase/capital letters
for signs/symbols (constants, variables, parameters)
denoting the same kind of object
is usually serviceable;
various examples are listed below

• in accordance with
the notational principle of Descartes,
it is often convenient & suggestive to use
the first part of the alphabet $a, b, c, \cdots$
for constants
&
the last part of the alphabet $\cdots, x, y, z$
for variables
&
to which could be added a suggestion
that the middle part of the alphabet,
a bit before and after m, n,
may sometimes be used for parameters
where a parameter
is understood to be a sign that
in a given discourse
may be sometimes considered to be a constant
and
may be sometimes considered to be a variable

• the general real first-degree/linear polynomial
in one variable
may be written

$$a x + b \qquad (a \neq 0)$$

wh a, b denote real numbers
& x is a real number variable

• the general real second-degree/quadratic polynomial
in one variable
may be written

$$a x^2 + b x + c \qquad (a \neq 0)$$

wh a, b, c denote real numbers
& x is a real number variable

• the general real third-degree/cubic polynomial
in one variable
may be written

$$a x^3 + b x^2 + c x + d \qquad (a \neq 0)$$

wh a, b, c, d denote real numbers
& x is a real number variable

• the general real
first-degree/linear polynomial equation
in two variables
may be written

$$A x + B y + C = 0 \quad (A \neq 0 \vee B \neq 0)$$

wh A, B, C denote real numbers
& x and y are real number variables

• the general real
second-degree/quadratic polynomial equation
in two variables
may be written

$$A x^2 + B x y + C y^2 + D x + E y + F = 0$$
$$(A \neq 0 \vee B \neq 0 \vee C \neq 0)$$

wh A to F denote real numbers
& x and y are real number variables

• note that
real variable = real number variable
complex variable = complex number variable

• sometimes it is possible
to start the run with the first letter of
the general name of
the object under consideration;
this notational device
may aid the memory

• f, g, h for functions
wh the run begins with
the first letter of ‘function’

• capital script ef $\mathcal{F}$, gee $\mathcal{G}$, aitch $\mathcal{H}$ for filters
wh the run begins with
the first letter of ‘filter’

• G, H, K for groups
wh the run begins with
the capitalized first letter of ‘group’
(with an understandable omission of I and J)

• i, j, k for indexes
wh the run begins with
the first letter of ‘index’

• $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ in bold-face letters for the basic tripod
of three mutually orthogonal unit 3-vectors;
bold-face emphasizes the distinction between
vectors & scalars

• p, q, r, s, t for propositions
wh the run begins with
the first letter of ‘proposition’

• p, q, r for primes
wh the run begins with
the first letter of ‘prime’

• X, Y, Z for topological spaces

• Greek letters are also highly useful;
eg
α, β, γ for angles
wh alpha corresponds to ay
and ay comes from ‘angle’

• the succession customarily used
may not always be
in alphabetic order
eg
ϕ, ψ, χ for functions
wh phi corresponds to ef
and ef comes from ‘function’;
again
ξ, η, ζ in place of or analogous to x, y, z
wh xi corresponds to ex
& eta corresponds to ee
& zeta corresponds to zee
in sound & transliteration

• a remarkable alphabetic run
of the last six letters of the alphabet
occurs in the traditional notation
of complex analysis
for the independent & dependent variables
of the complex function $w = f(z)$
viz

u, v, w, x, y, z

where

$u + iv = w = f(z)$
$x + iy = z$

whence
$u = u(x, y)$
$v = v(x, y)$

here
u, v, x, y are real number variables
&
w, z are complex number variables

Δ it may be advantageous to start with
some letter such as the first letter of the name
of the kind of object under study
and then use numerals or other symbols
as adscripts
to produce a sequence of symbols
eg

• the general real nth degree polynomial (n $\in$ pos int)
in one variable
may be written

$$a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x + a_n \qquad (a_0 \neq 0)$$

wh the coefficients are real numbers
& x is a real number variable

• $p_1, p_2, p_3, \cdots$
may be adopted as
a convenient sequence of proposition variables

• $a,\ a',\ a'',\cdots$
$b,\ b',\ b'',\cdots$
etc
wh the prime is used repetitively
may be adopted as
convenient signs
if no more than say three occurrences
of the prime are used

• the superscript in
$x^i\ (i = 1,\ 2,\ 3,\ \cdots\ )$
may be interpreted as
an index rather than an exponent
and thus give rise to
a sequence (finite or infinite) of variables;
in this case,
for example,
the square of
x^1
would be written
$(x^1)^2$

• if a capital letter say X is used to denote a set,
then the corresponding lowercase letter may be used
to start a run say x, y, z
or an indexed sequence say $x_1,\ x_2,\ x_3,\cdots$
to denote elements of the set

Δ the letter oh Oo, capital or lowercase,
is not to be recommended in general
for either constant or variable,
primarily because it looks so much like a zero 0;
however, capital oh O
serves admirably as the name of the origin
of a coordinate system
since oh is the first letter of ‘origin’
and is not likely to be confused with
any other symbol in this context;
also the origin O has coordinates
all equal to zero 0;
another possibility for the occasional use of capital oh O
is for an operation in prefix notation say
since oh is the first letter of ‘operation’;
$O(x_1, x_2, \cdots, x_n)$ looks good enuf;
what may look like a lowercase oh o
for the superscript degree sign as in 90°
is actually a small circle;
the suspended small circle is
also available to denote
a binary operation as say in the infix notation
$x \circ y$, read ‘x op y’,
wh ‘op’ comes from
the first two letters of ‘operation’

∆ letter styles often used for
constants & variables include:
• Roman
• boldface
• script
• open-face

∆ we do not need
a completed infinite totality
= an actual infinity
of signs for mathematics;
all we need is a potential infinity of signs
viz
no matter how many we use,
there is always one more available;
we state this notion in three languages below
• potential infinity
= always one more
= toujours encore un (French)
= toujours un de plus (French)
= immer ein mehr (German)
= immer noch ein (German)
• Immer noch ein Tröpfchen
aus dem kleinen Henkeltöpfchen
= lit: Always yet one droplet
out of the small handle jar
is a passage from
a bouncy German beer drinking song