The Early History of General Topological Dynamics

#12 of Gottschalk's Gestalts

A Series Illustrating Innovative Forms
of the Organization & Exposition
of Mathematics
by Walter Gottschalk

Infinite Vistas Press
PVD RI
2000

GG12-1 (14)

500 Angell St #414
Providence RI 02906

GG12-02

□ some personal and informal notes
on the early history of
general topological dynamics

Somewhere I read that G D Birkhoff once said that
if he thought mathematics exposition to be important,
he would be the world's best expositor.
He was certainly not the world's best expositor
and indeed he came close to the extremum
in the other direction.
I think this attitude had an important delaying effect
on the initial development of topological dynamics.
In his American Mathematical Society Colloquium volume
'Dynamical Systems' published in 1927
Birkhoff included a discussion
of the topological properties of continuous flows
determined by a system
of first order ordinary differential equations.
Actually his ideas on the subject go back much earlier to 1912.
The style of writing he adopted was so inadequate
in clarity and precision
that almost any beginning reader
had to be discouraged from continuing.
It was not at all clear what the theorems were
and the offered proofs
were largely suggestive intuitive discussions.

Now a clear precise systematic organization of this material
would require a major expenditure of time and energy.
And the recognition others would give to such ‘editorial’ work
may not be sufficient reward.
In other words, whatever someone did
to straighten out Birkhoff’s writing
would not be recognized to any extent
as at least partly his/her own accomplishment
but just a restatement of Birkhoff’s work.
However, reading and trying to understand Birkhoff is often
performing research on the basis of a few suggestions
and yet not be able to receive credit for the research.
Hence, it is not surprising that Birkhoff’s work
on flows in his Colloquium volume
remained virtually untouched for over a decade.

Sometimes an important mistake would occur in Birkhoff’s work,
just because of the style of exposition.
It must have been his natural insight
that kept him mostly on the path of provable theorems.
The topological language available
at the time of his Colloquium volume
was entirely adequate for his treatment of flows
but he preferred not to use it.
However, the topological situation was different in 1912
when he was beginning to think about these matters.

On occasion a mathematician will have an insight
that is ahead of the time in the sense that
the insight is not fully expressible
in the mathematical theory and language
developed at that moment.
For example, the Poincaré recurrence theorem
as first ‘stated and proved’ by Poincaré
was strictly not meaningful.
What was needed was
the language of Lebesgue measure which came later.
Much of the work of Élie Cartan in geometry and analysis
was done before the necessary supporting language
in algebra and topology appeared.
Of course, such a situation makes it
a forbiddingly difficult task to understand.
The best one can do is
to try to share the original intuitive insights,
which is not easy to come by.
And hope that no big mistakes are made.

Gustav Arnold Hedlund
came to the mathematics faculty
of the University of Virginia in 1939.
General topology was rampant at Virginia at the time.
The graduate students there
were an active research-minded group.
They were a primary source of inspiration to Arnold.
This situation likely contributed to
his thinking of writing up Birkhoff's work on flows
and other related work in the literature
not only from the point of view of
continuous flows on metric spaces
but also from the point of view of discrete flows
(generated by single homeomorphisms) on metric spaces;
the situation in which the flow came from
differential equations would then be
a special but particularly interesting case.

During his second year at Virginia,
Arnold composed and taught
a graduate course in topological dynamics
which treated in parallel fashion
firstly the theory for discrete flows on metric spaces
and then the theory for continuous flows on metric spaces.
Each theorem had two forms, stated and proved side by side.
The two forms were closely related but not exactly the same.
Clarity and precision were admirable.
I was a member of the class.
I think it would have been advantageous
to the mathematical community at that time
to have had those notes of his published
but he preferred not to do it.
Perhaps he did not want to spend the time
on writing for publication and overseeing the editorial details
because he wanted the time to spend
on his burgeoning research in this direction.

I devoted my Ph D dissertation under Arnold's supervision
to single continuous maps and single homeomorphisms
on topological spaces and metric spaces.
I completed the research in the summer of 1943.
I received my Ph D degree formally in February 1944.
Within a year afterward I began to realize that there was
a far-reaching theory for topological groups
acting as transformation groups
on topological spaces and uniform spaces
in which generality is to be optimized.
This theory would unify the study
of the two kinds of flows on metric spaces
and present a basis for a wide-ranging extension.
The study of such general objects
with respect to topological properties suggested originally
by the qualitative theory of ordinary differential equations
is to be called 'general topological dynamics'.
The term 'topological dynamics' was introduced by Arnold.
It was in contrast to the standard term 'analytical dynamics'.

Initially Arnold thought of topological dynamics
as concerning discrete and continuous flows on metric spaces.
Since the historical classes of examples,
symbolic dynamics and geodesic flows
(actually these two classes
of discrete flows and continuous flows
are themselves related),
are a major source of examples
and since Arnold was expert in both fields,
he was well motivated and well qualified
to undertake initial investigations in topological dynamics.

I felt in the middle 1940's that the theories of
transformation groups and transformation semigroups
would have something in common only at the beginning
and would soon diverge.
I think later developments justified that estimate.
I decided then to restrict my attention
mostly to transformation groups,
not because semigroups had no interest for me,
but because I could not do both as well as one
and because the transformation group hypothesis
was likely the first that should be considered in depth
for historical reasons and also for intrinsic reasons.
For example, the lack of symmetry in semigroups (no inverses)
made them intrinsically harder
in which to begin and sustain an investigation.
The theories are just different.

When Arnold began
to think about and to write his course notes
on the topological dynamics of flows about 1940,
he noticed that the basic terminology in the literature
had no uniformity.
The same word or phrase was used in different senses,
for example.
He settled on ‘recurrent’ and ‘almost periodic’
in the present day senses for flows.
I heartily agreed.

From time to time over the years
I selected some words and phrases
for certain objects and properties
relating to transformation groups.
For example: acting group, action, ambience, ambit, distal,
expansive, extensive, greatest ambit, inheritance, isochronous,
motion, phase space, proximal, recursive (at a point, pointwise,
uniformly, etc), replete, reversible/irreversible, substitution
minimal set, syndetic, totally minimal set, trace, transition,
universal minimal set, weakly almost periodic.
Often the notion itself was new to the literature.

The first papers on general topological dynamics
appeared in 1946.
So general topological dynamics is about fifty-five years old.
Have I received any surprises about its applications?
The applications to geometry and analysis that I know about,
I more or less expected.
But what did surprise me (very pleasantly indeed)
was the significant applications to number theory.
Fractals and the associated theory
would be classified under transformation semigroups.

Arnold was delighted at
the present-day great interest in symbolic dynamics.
Earlier he was almost the single practitioner of that specialty.
I would regard Arnold as
the lone de facto sustained founder of symbolic dynamics.
The initial stirring of symbolic dynamics
can be traced back to Hadamard and his work on geodesic flows
in 1898.
The Norwegian mathematician Thue
began a detailed combinatorial study of symbolic sequences
in 1906 without any reference to
geometry or analysis or topology,
but his work was not widely known early on
because he published in a journal of limited circulation.
A few others entered, dabbled, and left.
Arnold alone stayed until it flourished.
Much of his work in symbolic dynamics
was manifest in the supervision of the dissertations
of his Ph D students which involved the subject.
He also engaged in classified government work
which made heavy use of symbolic dynamics.
He talked to and had correspondence with
many mathematicians about the subject.
Arnold has never received quite the recognition
he deserves for his basic published work
and his personal influence in symbolic dynamics.
It was a bitter experience for him toward the last part of his life.

May I say that it was an honor and a privilege and a joy
to have had
a professional and personal association with Arnold
for a half-century.
Mathematics and the world would be better
if more people were like him.

What about the future?
I would expect the dynamical aspects
of transformation groups and semigroups
to continue to attract attention and to incite interest.

March 2000