Math Snippets: Sixth Bouquet

#96 of Gottschalk's Gestalts

A Series Illustrating Innovative Forms
of the Organization & Exposition
of Mathematics
by Walter Gottschalk

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☐ ¿why
numerator & denominator
of a fraction?

• numerator (English noun)
= one who/that which
numbers
↑
to numerate (English verb)
= to give a number to
↑
numeratus (Latin verb, past participle)
= numbered
numerare (infinitive)
= to number
↑
numerus (Latin noun, nominative case)
= number
↑
*nem- (IE root)
= to

• denominator (English noun)
= one who/that which
denominates
↑
to denominate (English verb)
= to give a name to
↑
denominatus (Latin verb, past participle)
= gave a name to
denominare (infinitive)
= to give a name to
↑
de (Latin preposition)
= from
+
nominare (Latin verb, infinitive)
= to name
↑
nominis (Latin noun, genitive case)
nomen (nominative case)
= name
↑
*nomen (IE root)
= name

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• a denominate number
= df a number together with a unit of measure
which is called
the denomination
of the denominate number

• for a fraction
the numerator gives the number of times
the denominator is to be taken
ie
the numerator gives the numeration
&
the denominator gives the denomination;
thus

three fourths $= \frac{3}{4} = 3 \times \frac{1}{4} =$ three times a fourth

• fraction (English noun)
↑
fractionem (Latin noun, accusative case)
fractio (nominative case)
= fracture/breaking
↑
fractus (Latin verb, past participle)
= broken
frangere (infinitive)
= to break
↑
*bhreg- (IE root)
= to break

☐ The Global Gauss - Bonnet Theorem

(1) entirely in words

the integrated Gaussian curvature of a closed surface

equals

two pi times the Euler characteristic

(2) entirely in symbols

$$\int_S K\,dA = 2\pi\chi(S)$$

☐ to cut the Gordian knot

Δ this is mostly a pleasant fable
altho
the knot was real,
Alexander was real,
some relationship between the two did in fact exist,
the geography was real

• Gordius was a peasant of Phrygia,
an ancient country in west central Asia Minor,
& the father of Midas of the gold-converting touch;
Asia Minor = the peninsula forming
a western extremity of Asia
and bounded on three sides by
the Black Sea on the north,
the Mediterranean Sea on the south,
the Aegean Sea on the west;
Asia Minor contains the greater part of
present-day Asiatic Turkey

• when Gordius was chosen king
on the advice of an oracle,
he dedicated a chariot to Zeus
& fastened the yoke to the pole
using a rope of bark
in such an ingenious & complicated way
including hidden ends
that no one could untie the knot

• Alexander the Great
356-323 BCE
King of Macedon
(Macedon was an ancient country
including part of northern present-day Greece)
&
conqueror of most of the known world,
was told in 333 BCE on his march thru Asia
that
'whoever undid the knot
would reign over the whole East';
Alexander replied
'It is thus that I perform this task!'
&
cut the knot in two with his sword

Δ hence

• a Gordian knot
= df a difficult/intricate problem/situation

• to cut the Gordian knot
= df
(1) to get out of a difficult situation
by one decisive step
or
(2) to resolve a difficult situation
by force or by evasive action
or
(3) to solve a difficult & intricate problem
by a surprising & simple means

☐ notation for logarithms

• the real logarithms
to the bases 10 and e
of a positive real number x

the common logarithm of x
$=_{dn}$ lc x wh l ← logarithm & c ← common
$=_{rd}$ com log (of) x
$=_{df}$ $\log_{10} x$ (sv)

the natural logarithm of x
$=_{dn}$ ln x wh l ← logarithm & n ← natural
$=_{rd}$ nat log (of) x
$=_{df}$ $\log_e x$ (sv)

• the complex logarithms
to the base e
of a nonzero complex number z

the multivalued complex logarithm of z
$=_{dn}$ $\log z$ wh log ← logarithm
$=_{rd}$ (mv comp) log (of) z
$=_{df}$ $\ln|z| + i \operatorname{ang} z$ (mv)

the principal - valued complex logarithm of z
$=_{dn}$ $\operatorname{Log} z$ wh Log ← logarithm
$=_{rd}$ (pv comp) log (of) z
$=_{df}$ $\ln|z| + i \operatorname{Ang} z$ (sv)

wh
$\operatorname{Ang} z \in Qz$
$-\pi < \operatorname{Ang} z \leq \pi$
$\operatorname{ang} z = \operatorname{Ang} z + 2\pi\mathbb{Z}$

☐ the goals of alchemy

the four main goals of medieval alchemy
were to find:

• the philosopher's stone
= lapis philosophorum (Latin)
= a substance that permits
the transmutation of base metals
such as iron or lead
into gold

• the universal solvent
= alkahest (English)
= alchahest (Latin, first used by Paracelsus (1493-1541)
and said to have been coined by him
in imitation of Arabic words)
= a liquid that dissolves everything
(but what do you keep it in?)

• the panacea
= panacea (Latin)
= a substance that cures all human illness

• the elixir of ife
= elixir vitae (Latin)
= a substance that prolongs human life indefinitely

the alchemists' unremitting tho unsuccessful search
for these things
laid the foundation
for the modern science of chemistry;
alchemy was recognized as
a serious scholarly endeavor
from about the beginning of the common era
to about 1700;
two notable figures that engaged in alchemy were
Roger Bacon
ca 1214-1292
English
&
Isaac Newton
1642-1727
English

¿what is the philosopher's stone of mathematics?

□ related curves & surfaces

- catenary
& catenoid

- ellipse
& ellipsoid

- hyperbola
& hyperboloid

- parabola
& paraboloid

- helix
& helicoid

- oval
& ovoid

- circle
& circular cylinder

- ellipse
& elliptic cylinder

- hyperbola
& hyperbolic cylinder

- parabola
& parabolic cylinder

☐ related sufaces

- cone
& conoid

- cylinder
& cylindroid

- sphere
& spheroid

- oblate ellipsoid
& prolate ellipsoid

- hyperboloid of one sheet
& hyperboloid of two sheets

- hyperboloid of revolution of one sheet
& hyperboloid of revolution of two sheets

□ the two simplest infinite ambits
express dynamically
the axioms for the natural numbers
&
the axioms for the integers

Δ consider the ordered triple
(X, a, φ)
wh
- $X \in$ set
- $a \in X$
- $\varphi : X \to X \in 1-\text{to}-1$

Δ the semigroup case

let

- $a \notin X\varphi$
- $\forall E \subset X . a \in E \,\&\, E\varphi \subset E \Rightarrow E = X$

which is the mathemathical induction axiom

&

which states in dynamical terms:

the only subinvariant set

that contains the base point is the space

then

- (X, a, φ) is uniquely isomorphic to

the natural number shift system $(\mathbb{N},\ 0,\ n \mapsto n+1)$

Δ the group case

let

- $X\varphi = X$
- $\forall E \subset X . a \in E \;\&\; E\varphi = E \Rightarrow E = X$

which is the bilateral mathematical induction axiom

&

which states in dynamical terms:

the only invariant set

that contains the base point is the space

then

- (X, a, φ) is uniquely isomorphic to

the integer shift system $(\mathbb{Z},\ 0,\ n \mapsto n+1)$

☐ the minimalist doctrne of
less-is-more
may be symbolized by the incorrect statement
< = >

• 'less is more'was first stated by
Robert Brownng
1812-1889
English
poet
in his poem
'Andrea del Sarto' (1855)

• the minimalist doctrine of less-is-more
was subscribed to by various minimalist
architects, painters, sculptors
of the earlier part of the 20th century

• the opposing maximalist doctrine
may be called
more-is-better
&
may be symbolized by the nonsense statement
> = +

• ¿do either of these two opposng doctrines
have relevance to mathematics?

☐ algebraic sum rule for derivatives

let

- $u_i = u_i(x) \in C^1$ for $i \in \underline{n}$ wh $n \in$ pos int
- $e_i \in \{1, -1\}$ for $i \in \underline{n}$

then

- $$\frac{d}{dx}\sum_{i=1}^{n} e_i u_i = \sum_{i=1}^{n} e_i \frac{du_i}{dx}$$

- derivative of algebraic sum
  equals
  algebraic sum of derivatives

□ to differentiate a product
differentiate each factor separately
and add

$$(uv)' = u'v + uv'$$

$$(uvw)' = u'vw + uv'w + uvw'$$

etc

☐ ¿what is Ada's full name?
bioline
Lady Augusta Ada Byron King, Countess of Lovelace
1815-1852
English
writer, mathematician, socialite;
the first computer programmer;
assistant and patron of Charles Babbage;
she wrote programs for his 'Analytical Engine',
commonly considered the first computer;
in 1980 the high-level universal
computer programming language ADA
was named in her honor

• daughter of the married couple,
the English poet Lord Byron
& Annabella Milbanke;
her birth name was
Augusta Ada Byron
and being the daughter of a Baron
had the title of Lady;
in 1835 she wed William King;
in 1838 they became Earl and Countess of Lovelace;
her name was then in full
Lady Augusta Ada Byron King, Countess of Lovelace;
her name is often shortened to Ada Lovelace

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