Math Snippets: Fifth Bouquet

#95 of Gottschalk's Gestalts

A Series Illustrating Innovative Forms
of the Organization & Exposition
of Mathematics
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Infinite Vistas Press
PVD RI
2003

GG95-1 (25)

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GG95-2

□ semiphilosophical comments on ordinals & sequences

• the most general notion of sequence
is that of a function on an ordinal

• the notions of ordinal & sequence
are the mathematical embodiments
of the general intuitive notion of ‘next’;
the difference is that
for an ordinal all the terms are distinct
&
for a sequence the terms may not be distinct

• an ordinal is (the measure of) the length of a sequence

☐ bioline
Ahmes the Scribe
fl ca 1650 BCE
Egyptian

the Rhind Mathematical Papyrus
was bought in 1856 in a Nile resort town
by a Scottish antiquary Henry Rhind;
the papyrus was written by the Egyptian scribe Ahmes
about 1650 BCE who states that it is based on a prototype
from the Middle Kingdom ca 2000–1800 BCE;
the papyrus concerns arithmetical procedures
and the solution of mundane day-to-day problems
by arithmetical calculations;
the Rhind Mathematical Papyrus
could be considered to be
the first extant book on mathematics;
here is a free translation of its title
from the hieratic and hieroglyphic texts:

Accurate reckoning.
The entrance into the knowledge
of all existing things and all obscure secrets.

□ in a field

• addition is a composite of subtraction and negation

$$a + b = a - (-b)$$

• subtraction is a composite of addition and negation

$$a - b = a + (-b)$$

• multiplication is a composite of division and reciprocation

$$ab = a \div b^{-1} \quad (b \neq 0)$$

• division is a composite of multiplication and reciprocation

$$a \div b = ab^{-1} \quad (b \neq 0)$$

□ the definer $=_{df}$

is a compound symbol

consisting of an equality sign =

and a right - hand subscript dee ef

which suggests the word ' define'

&

means

' is defined to be'

or

' is defined to mean'

whichever is appropriate

a definition has the form

definiendum	definer	definiens
↑	↑	↑
that which is being defined = the defined	that which defines = the definer	that which serves to define = the defining

the definiendum
is defined to be / mean
the definiens

definiendum $=_{df}$ definiens

if subscripts are difficult,
an alternative notation would be
definiendum = df definiens

□ in general

an object $\neq$ a name of an object;

however

the use of a symbolic expression as a name for itself

often occurs and is called

autonomy (noun)

and

autonymous (adjective)

from the Greek word meaning 'self - naming'

an example of autonymy is:

f(x) contains a pair of parentheses

the usual mathematical exposition

has many autonymous instances

□ historical note on tensor analysis

The beginning notions of tensor analysis were implicit in the work, mostly differential geometry, of Gauss, particularly Riemann, and other 19th century mathematicians. Tensor analysis was organized and developed as an independent subject first by Ricci from 1892 on and then by Ricci & Levi-Civita from 1901 on. Ricci originally called tensor analysis 'the absolute differential calculus'. Tensor analysis is a notational system of relative brevity and great efficiency that allows some complicated things to be said in manageable ways. Uses of tensor analysis appeared early on in n-dimensional differential geometry and mathematical physics but it was not widely known or widely used in the first decade and a half of the 20th century. Einstein's general theory of relativity, annnounced in 1916, was a dramatic use of tensor analysis and stimulated great interest in tensor analysis. Einstein was taught differential geometry and tensor analysis by a mathematician colleague. Einstein in 1916 was the first to use the word 'tensor' in the present context and thus the name of the subject was changed to 'tensor analysis'. Differential geometry, using the language of tensor analysis, provides the mathematical foundation for the general theory of relativity.

The English noun ‘tensor’ is from the Latin verb ‘tendere’ meaning ‘to stretch’, which is from the Greek verb τεινω meaning ‘to stretch’. The word ‘tensor’ has long been used in the anatomy of muscles and in the mathematical theory of quaternions in senses appropriate to the above meaning in the classical languages. Einstein in 1916 was the first to use the word ‘tensor’ in its customary modern meaning in analysis, taking the word ‘tensor’ from the theory of elasticity where it means a second-order tensor in the context of ‘stress tensor’. Note that ‘stress’ and ‘tension’ can be synonyms. ‘A tensor causes tension.’

☐ historical note on determinants

• about 250 BCE Chinese mathematicians anticipated the notion and theory of determinants in solving systems of linear equations

• in 1683 the Japanese mathematician Seki Kowa had the idea of determinants and their expansion

• in 1693 the German mathematician Gottfried Wilhelm Leibniz in a letter to the French mathematician Guillaume L'Hôpital presented the notion of determinant in the form of its combinatorial definition, based on rows and columns of numbers used in the solution of a system of linear equations, but Leibniz's discovery was without influence; this marks the beginning of the theory of determinants in the Western world

• in 1750 the Swiss mathematician
Gabriel Cramer rediscovered
the combinatorial definition of determinant
in connection with the solution
of a system of linear equations;
he stated Cramer's Rule
but he was not the first to do so;
thereafter determinants came into general use

• in 1771 the French mathematician
Alexandre-Théophile Vandermonde gave the first
connected exposition of the theory of determinants and
may be therefore regarded as the formal founder
of the theory of determinants

• in 1801 the German mathematician
Carl Friedrich Gauss introduced
the word 'determinant'
but not in its present sense

• in 1812 the French mathematician
Augustin Louis Cauchy used
the word 'determinant' in its present sense

• in1841 the English mathematician
Arthur Cayley introduced the vertical bar notation
for determinants

□ the constant of Catalan

= the Catalan constant

= Catalan's constant

$=_{dn}$ ¢ = the cent sign = slashed lowercase cee

$=_{rd}$ kat

$$=_{df} \frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \cdots$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)^2}$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{x}{\sin x}\,dx = \frac{1}{2}\int_0^{\infty} \frac{x}{\cosh x}\,dx$$

$$= 0.91596\ 55941\ 77219+$$

it is not known (2002) whether ¢ is irrational

□ the identity of Lagrange

= the Lagrange identity

= Lagrange's identity

let

- R ∈ com ring
- n ∈ pos int
- $x_i, y_i \in R \quad (i \in \underline{n})$

then

- $$\left(\sum_{i=1}^{n} x_i^{\,2}\right)\left(\sum_{i=1}^{n} y_i^{\,2}\right)-\left(\sum_{i=1}^{n} x_i y_i\right)^2$$

$$= \sum_{\substack{i,j=1 \\ i<j}}^{n} (x_i y_j - x_j y_i)^2$$

$$= \frac{1}{2}\sum_{i,j=1}^{n} (x_i y_j - x_j y_i)^2 \quad \text{if } R \in \text{field of ch} \neq 2$$

□ John Bernoulli's integral & series

$$\int_0^1 x^x \, dx$$

$$= \frac{1}{1^1} - \frac{1}{2^2} + \frac{1}{3^3} - \frac{1}{4^4} + \cdots$$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^n}$$

$$= 0.78343\cdots$$

□ Dirichlet's discontinuous factor integral

$$\int_0^{\infty} \frac{\sin ax \cos bx}{x}\, dx \quad \text{wh } a,\ b\ \in\ \text{pos real nr}$$

$$= \ \frac{\pi}{2} \quad \text{if } a > b$$

&

$$= \ \frac{\pi}{4} \quad \text{if } a = b$$

&

$$= \ 0 \quad \text{if } a < b$$

□ a particularly pleasing pattern
of intensely interesting integrals

$$\int_0^\infty \frac{1}{1+x}\,dx = +\infty$$

$$\int_0^\infty \frac{1}{1+x^2}\,dx = \frac{1}{2}\pi$$

$$\int_0^\infty \frac{1}{1+x^3}\,dx = \frac{2}{9}\sqrt{3}\,\pi$$

$$\int_0^\infty \frac{1}{1+x^4}\,dx = \frac{1}{4}\sqrt{2}\,\pi$$

$$\int_0^\infty \frac{1}{1+x^5}\,dx = \frac{1}{25}\sqrt{50+10\sqrt{5}}\,\pi$$

$$\int_0^\infty \frac{1}{1+x^6}\,dx = \frac{1}{3}\pi$$

$$\int_0^\infty \frac{1}{1+x^n}\,dx = \frac{\pi}{n}\csc\frac{\pi}{n} \quad (2 \le n \in \text{int})$$

□ a wild guess or two
on the origin of the word ' radian'

radius / radial + angle
= radi + an by abbreviation
= radian by juxtaposition

again

radial − l + n
= radian
wh
l ← first (numeral one = 1) or last (initial letter el = l)
for first / last used angle measure
viz the sexagesimal angle measure of degrees etc
&
n ← new angle measure

☐ the three ways of doing science

there are now three ways
of conducting scientific research;
the first two ways are old;
the third way is new;
they are:

(1) the
inductive
experiential
experimental
obsevational
pragmatic
method
which was first clearly described by
Francis Bacon
1561-1626
Englsh
philosopher, statesman

(2) the
deductive
mathematical
rational
theoretical
method
which was first clearly described by
René Descartes
1596-1650
French
mathematician, philosopher;
father of modern philosophy

(3) the
computer computation
computer graphics
computer simulation
method
which was made possible
by the modern high-speed electronic computer

in general
these ways may be substantially intertwined

☐ the six levels
at which science is communicated

(1) personal science

(2) internet science

(3) journal science

(4) handbook science

(5) textbook science

(6) popular science

☐ the scientific progression
with mounting
evidence/verification
from
cognition/experiment/observation/prediction

the question
↓
speculation
↓
conjecture
↓
hypothesis
↓
theory = the accepted answer

□ the goal of physics
= the gold medal of physics

physics seeks
an ultimate final law
that will describe one symmetric structure
which unifies
all elementary forces
&
all elementary particles

= the holy grail of physics
= HGP

= the grand unification/unified theory
= GUT

¿what is HGM = the holy grail of mathematics?

□ the anthropic cosmological principle
= ACP
= the doctrine that
the existence of humankind
accounts for at least some of
the characteristics
of the universe around us
wh
anthropic
= of mankind

cosmological
= pertaining to cosmology

cosmology
= the study of the universe as a whole
& particularly in its early history

¿does ACP have any meaning for mathematics?

☐ the number of possible positions

- in checkers $\approx 10^{20}$
- in chess $\approx 10^{44}$
- in Go $\approx 10^{120}$