Some Quaint & Curious & Almost Forgotten Trig Functions

#80 of Gottschalk’s Gestalts

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GG80-1 (25)
six related trigonometric functions, antiquated & quaint & curious, that are primarily of historical interest, with some still sometimes somewhat useful.

let

• \( A \in \text{angle} \)

then

• the versed sine of \( A \)

\[
=_{ab} \text{the versine of } A
\]

\[
=_{dn} \text{vers } A
\]

\[
=_{rd} \text{ver - sine } A
\]

\[
= \text{verse } A
\]

\[
=_{df} 1 - \cos A
\]

wh

versine \( \leftarrow \) versed sine

vers \( \leftarrow \) versine

versed = turned

GG80-3
• the covered sine of $A$
  = the versed cosine of $A$
  =$_{ab}$ the coversine of $A$
  =$_{dn}$ covers $A$
  =$_{rd}$ koh-ver-sine $A$
  = koh-verse $A$
  =$_{df}$ $1-\sin A$

wh

covered sine $\leftarrow$ versed sine of complement

coversine $\leftarrow$ covered sine

covers $\leftarrow$ coversine

note that

the versed sine of $A$
  = $1-\cos A$

&

the versed cosine of $A$
  = $1-\sin A$

GG80-4
• half of the versed sine of A
  = half of the versine of A
  = \( \frac{1}{2} \) vers A
  = \( \frac{1}{2} (1 - \cos A) \)
  = hav A
  = havers A
  = haversine A
  = hav - er - sine A
  = hav - erse A
  = have A
  = \( \frac{1}{2} \) vers A

wh
haversine \( \leftarrow \) half of versine
havers \( \leftarrow \) haversine
hav \( \leftarrow \) haversine
• half of the covered sine of A
  = half of the versed cosine of A
  = half of the coversine of A
  =_{ab} hacoversine of A
  =_{dn} hacovers A
  =_{rd} hack-o-ver-sine A
  = hack-o-verse A
  =_{df} \frac{1}{2} \text{covers A}
  = \frac{1}{2}(1 - \sin A)

wh

hacoversine ← half of coversine
hacovers ← hacoversine
• the external secant of A
  \( =_{ab} \) the exsecant of A
  \( =_{dn} \) exsec A
  \( =_{rd} \) ecks - see - cant A
  \( = \) ecks - seck A
  \( =_{df} \) sec A \( - 1 \)

wh

exsecant \( \leftarrow \) external secant

exsec \( \leftarrow \) exsecant
• the external cosecant of A
  \[ a_b \] the excosecant of A
  \[ d_n \] excsc A
  \[ r_d \] ecks-koh-see-cant A
  \[ = ecks-koh-seck A \]
  \[ d_f \] csc A – 1


wh

excosecant ← external cosecant
excsc ← excosecant
Keep the identities involving these trig fcns

• vers A = 1 – cos A = 2 \sin^2 \frac{A}{2}

• covers A = 1 – sin A

• havers A = \frac{1}{2} (1 – \cos A) = \sin^2 \frac{A}{2}

• hacovers A = \frac{1}{2} (1 – \sin A)

• exsec A = \sec A – 1

• excsc A = \csc A – 1
• vers $A = 2 \text{havers } A$

• covers $A = 2 \text{covers } A$

• havers $A = \frac{1}{2} \text{vers } A$

• hacovers $A = \frac{1}{2} \text{covers } A$

• exsec $A = \text{sec } A \text{ vers } A$

• excsc $A = \text{csc } A \text{ covers } A$
• vers $\hat{A}$ = covers $A$

• covers $\hat{A}$ = vers $A$

• havers $\hat{A}$ = hacovers $A$

• hacovers $\hat{A}$ = havers $A$

• exsec $\hat{A}$ = excsc $A$

• excsc $\hat{A}$ = exsec $A$

wh
$\hat{A}$

$\hat{A}$ = _rd_ comp $A$ = $A$ comp

$\hat{A}$ = _df_ the complement of $A$

comp $\leftarrow$ complement

note the overscript suggests
a right angle opening downward

GG80-11
• \( \text{vers } \overline{A} = 2 - \text{vers } A \)

• \( \text{covers } \overline{A} = \text{covers } A \)

• \( \text{havers } \overline{A} = 1 - \text{havers } A \)

• \( \text{hacovers } \overline{A} = \text{hacovers } A \)

• \( \text{exsec } \overline{A} = -2 - \text{exsec } A \)

• \( \text{excsc } \overline{A} = \text{excosec } A \)

wh

\( \overline{A} \)

=_{\text{rd}} \text{sup } A = \text{A sup}

=_{\text{df}} \text{the supplement of } A

sup \leftarrow \text{supplement}

note the overscript suggests

a straight angle

GG80-12
• vers (−A) = vers A

• covers (−A) = 2 − covers A

• havers (−A) = havers A

• hacovers (−A) = 1 − hacovers A

• exsec (−A) = exsec A

• excsc (−A) = −2 − excsc A
• vers $A \geq 0$

• covers $A \geq 0$

• havers $A \geq 0$

• hacovers $A \geq 0$
the haversine formula for the angles of a plane triangle

- \( \text{hav } A = \frac{(s - b)(s - c)}{bc} \)

& cyclically

the haversine formula for the angles of a spherical triangle

- \( \text{hav } A \)
  \[ \quad = \frac{\sin(s - b) \sin(s - c)}{\sin b \sin c} \]
  \[ \quad = \frac{\text{hav } a - \text{hav } (b - c)}{\sin b \sin c} \]
  \[ \quad = \text{hav}[\pi - (B + C)] + \sin B \sin C \text{ hav } a \]

& cyclically

GG80-15
the haversine formula for the sides of a spherical triangle

\[ \text{hav} a = \text{hav} (b - c) + \sin b \sin c \text{ hav} A \]

& cyclically

note: this formula may be used to find the great-circle distance and the bearing between two positions on the Earth's surface once their latitude & longitude are known
in times gone by
viz
in the 18th & the 19th & the early part of the 20th centuries
these trig functions
were used rather frequently
in geography & in marine navigation;
even today you may see
some appearance of some of them
& not only in matters involving
the history of mathematics
a study of the following labeled diagrams will reveal reasons for the designations of the four trig functions

• the versed sine of A
  = vers A
  = 1 – cos A

• the covered sine of A
  = covers A
  = 1 – sin A

• the exsecant of A
  = exsec A
  = sec A – 1

• the excosecant of A
  = excsc A
  = csc A – 1

GG80-18
etymology

- sinus rectus (Latin, historical term)
  = vertical sine
  = sine

- sinus versus (Latin, historical term)
  = versed sine
  = sine turned on its side
  = versine

- coversine
  = versine of complement

- exsecant
  = external part of the secant

- excosecant
  = external part of the cosecant

GG80-19
covers A

sin A
\[ \text{csc } A \quad \text{excsc } A \]
identify the line segments representing \( \text{ie} \) whose lengths are

\[
\begin{align*}
\sin A &= \cos \hat{A} \\
\cos A &= \sin \hat{A} \\
\tan A &= \cot \hat{A} \\
\cot A &= \tan \hat{A} \\
\sec A &= \csc \hat{A} \\
\csc A &= \sec \hat{A} \\
\vers A &= \covers \hat{A} \\
\covers A &= \vers \hat{A} \\
\exsec A &= \excsc \hat{A} \\
\excsc A &= \exsec \hat{A}
\end{align*}
\]

GG80-25