Some Quaint & Curious & Almost Forgotten Trig Functions

#80 of Gottschalk's Gestalts

A Series Illustrating Innovative Forms
of the Organization & Exposition
of Mathematics
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□ six related trigonometric functions,
antiquated & quaint & curious,
that are primarily of historical interest,
with some still sometimes somewhat useful

let

• A $\in$ angle

then

• the versed sine of A

$=_{ab}$ the versine of A

$=_{dn}$ vers A

$=_{rd}$ ver - sine A

= verse A

$=_{df}$ $1 - \cos A$

wh

versine $\leftarrow$ <u>ver</u>sed <u>sine</u>

vers $\leftarrow$ <u>vers</u>ine

versed = turned

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• the coversed sine of A

$=$ the versed cosine of A

$=_{ab}$ the coversine of A

$=_{dn}$ covers A

$=_{rd}$ koh - ver - sine A

$=$ koh - verse A

$=_{df}$ $1 - \sin A$

wh

coversed sine ← versed sine of complement

coversine ← coversed sine

covers ← coversine

note that

the versed sine of A

$= 1 - \cos A$

&

the versed cosine of A

$= 1 - \sin A$

• half of the versed sine of A

$=$ half of the versine of A

$=_{ab}$ haversine of A

$=_{dn}$ havers A

$=$ hav A

$=_{rd}$ hav - er - sine A

$=$ hav - erse A

$=$ have A

$=_{df} \frac{1}{2}$ vers A

$= \frac{1}{2}(1 - \cos A)$

wh

haversine $\leftarrow$ half of versine

havers $\leftarrow$ haversine

hav $\leftarrow$ haversine

• half of the coversed sine of A

= half of the versed cosine of A

= half of the coversine of A

$=_{ab}$ hacoversine of A

$=_{dn}$ hacovers A

$=_{rd}$ hack - o - ver - sine A

= hack - o - verse A

$=_{df} \frac{1}{2}$ covers A

$= \frac{1}{2}(1 - \sin A)$

wh

hacoversine ← half of coversine

hacovers ← hacoversine

• the external secant of A

$=_{ab}$ the exsecant of A

$=_{dn}$ exsec A

$=_{rd}$ ecks - see - cant A

= ecks - seck A

$=_{df}$ $\sec A - 1$

wh

exsecant ← external secant

exsec ← exsecant

• the external cosecant of A

$=_{ab}$ the excosecant of A

$=_{dn}$ excsc A

$=_{rd}$ ecks - koh - see - cant A

= ecks - koh - seck A

$=_{df}$ $\csc A - 1$

wh

excosecant ← ex̲t̲ernal c̲o̲s̲e̲c̲a̲n̲t̲

excsc ← e̲x̲c̲os̲e c̲ant

□ some identities involving these trig fcns

- vers A $= 1 - \cos A = 2\sin^2 \frac{A}{2}$

- covers A $= 1 - \sin A$

- havers A $= \frac{1}{2}(1 - \cos A) = \sin^2 \frac{A}{2}$

- hacovers A $= \frac{1}{2}(1 - \sin A)$

- exsec A $= \sec A - 1$

- excsc A $= \csc A - 1$

- $\text{vers}\,A = 2\,\text{havers}\,A$
- $\text{covers}\,A = 2\,\text{covers}\,A$
- $\text{havers}\,A = \frac{1}{2}\text{vers}\,A$
- $\text{hacovers}\,A = \frac{1}{2}\text{covers}\,A$
- $\text{exsec}\,A = \sec A\,\text{vers}\,A$
- $\text{excsc}\,A = \csc A\,\text{covers}\,A$

• vers Â = covers A

• covers Â = vers A

• havers Â = hacovers A

• hacovers Â = havers A

• exsec Â = excsc A

• excsc Â = exsec A

wh

Â

$=_{rd}$ comp A = A comp

$=_{df}$ the complement of A

comp ← complement

note the overscript suggests

a right angle opening downward

• $\operatorname{vers} \overline{A} = 2 - \operatorname{vers} A$

• $\operatorname{covers} \overline{A} = \operatorname{covers} A$

• $\operatorname{havers} \overline{A} = 1 - \operatorname{havers} A$

• $\operatorname{hacovers} \overline{A} = \operatorname{hacovers} A$

• $\operatorname{exsec} \overline{A} = -2 - \operatorname{exsec} A$

• $\operatorname{excsc} \overline{A} = \operatorname{excosec} A$

wh

$\overline{A}$

$=_{rd}$ sup A = A sup

$=_{df}$ the supplement of A

sup ← supplement

note the overscript suggests

a straight angle

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- $\text{vers}(-A) = \text{vers} A$
- $\text{covers}(-A) = 2 - \text{covers} A$
- $\text{havers}(-A) = \text{havers} A$
- $\text{hacovers}(-A) = 1 - \text{hacovers} A$
- $\text{exsec}(-A) = \text{exsec} A$
- $\text{excsc}(-A) = -2 - \text{excsc} A$

- vers A $\geq$ 0
- covers A $\geq$ 0
- havers A $\geq$ 0
- hacovers A $\geq$ 0

□ the haversine formula
for the angles of a plane triangle

• $\text{hav } A = \dfrac{(s-b)(s-c)}{bc}$

& cyclically

□ the haversine formula
for the angles of a spherical triangle

• hav A

$$= \frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}$$

$$= \frac{\text{hav } a - \text{hav}(b-c)}{\sin b \sin c}$$

$$= \text{hav}\left[\pi - (B+C)\right] + \sin B \sin C \,\text{hav } a$$

& cyclically

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□ the haversine formula
for the sides of a spherical triangle

• $\text{hav}\, a = \text{hav}(b - c) + \sin b \sin c \,\text{hav}\, A$

& cyclically

note: this formula may be used to find the great - circle distance and the bearing between two positions on the Earth's surface once their latitude & longitude are known

□ in times gone by
viz
in the 18th & the 19th & the early part of the 20th
centuries
these trig functions
were used rather frequently
in geography & in marine navigation;
even today you may see
some appearance of some of them
& not only in matters involving
the history of mathematics

□ a study of the following
labeled diagrams
will reveal reasons for
the designations of the four trig functions

- the versed sine of A

$= \text{vers } A$

$= 1 - \cos A$

- the coversed sine of A

$= \text{covers } A$

$= 1 - \sin A$

- the exsecant of A

$= \text{exsec } A$

$= \sec A - 1$

- the excosecant of A

$= \text{excsc } A$

$= \csc A - 1$

☐ etymology

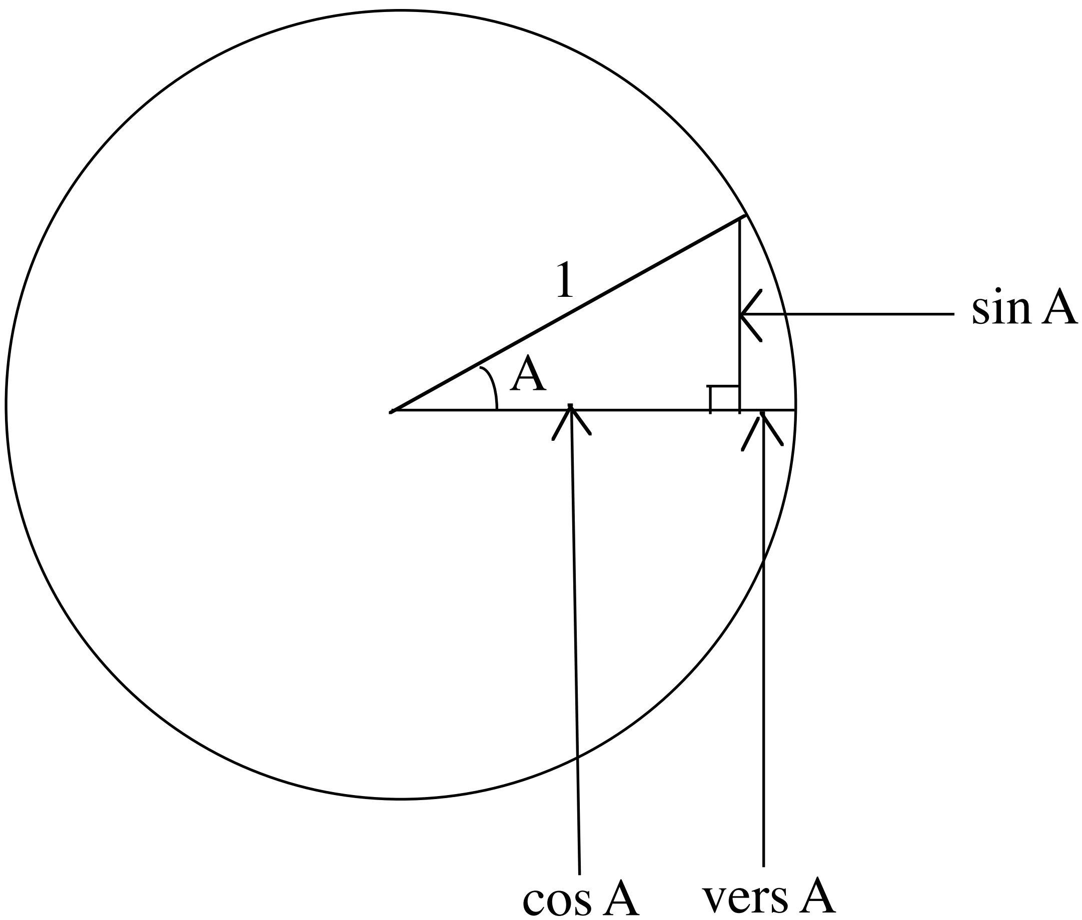
• sinus rectus (Latin, historical term)
= vertical sine
= sine

• sinus versus (Latin, historical term)
= versed sine
= sine turned on its side
= versine

• coversine
= versine of complement

• exsecant
= external part of the secant

• excosecant
= external part of the cosecant



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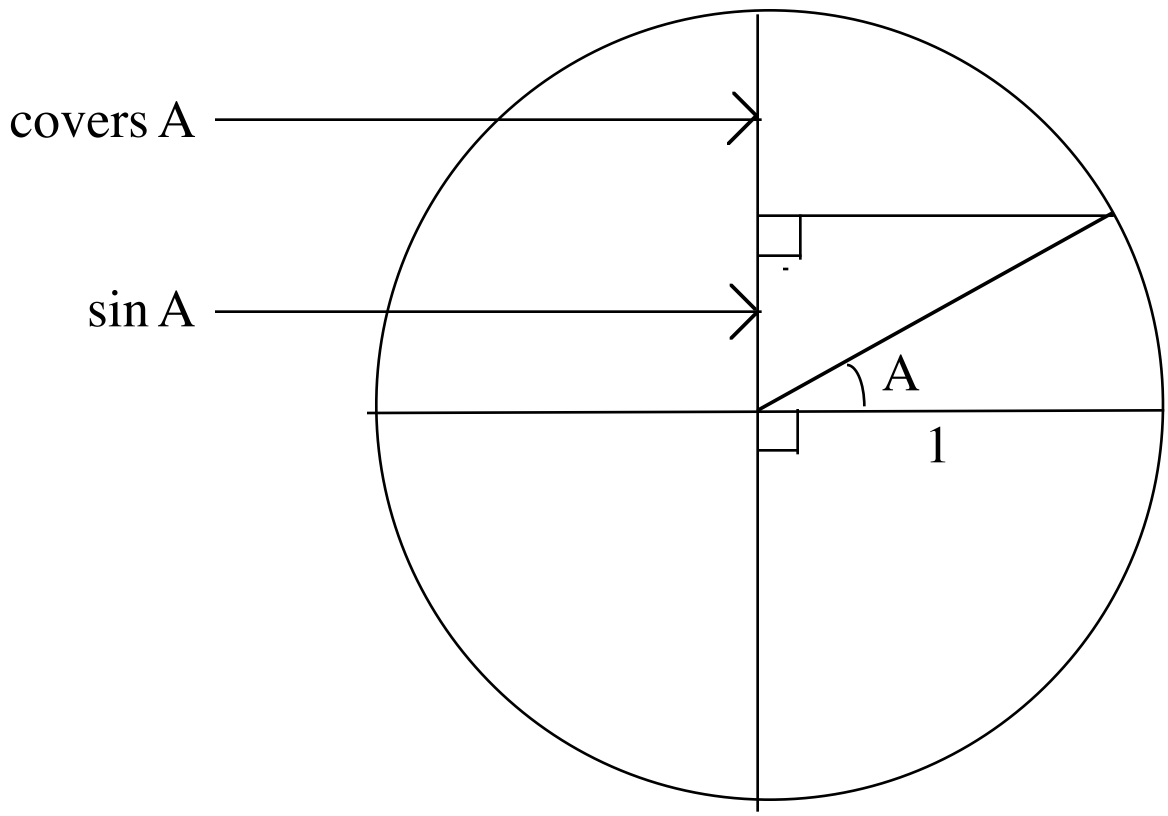

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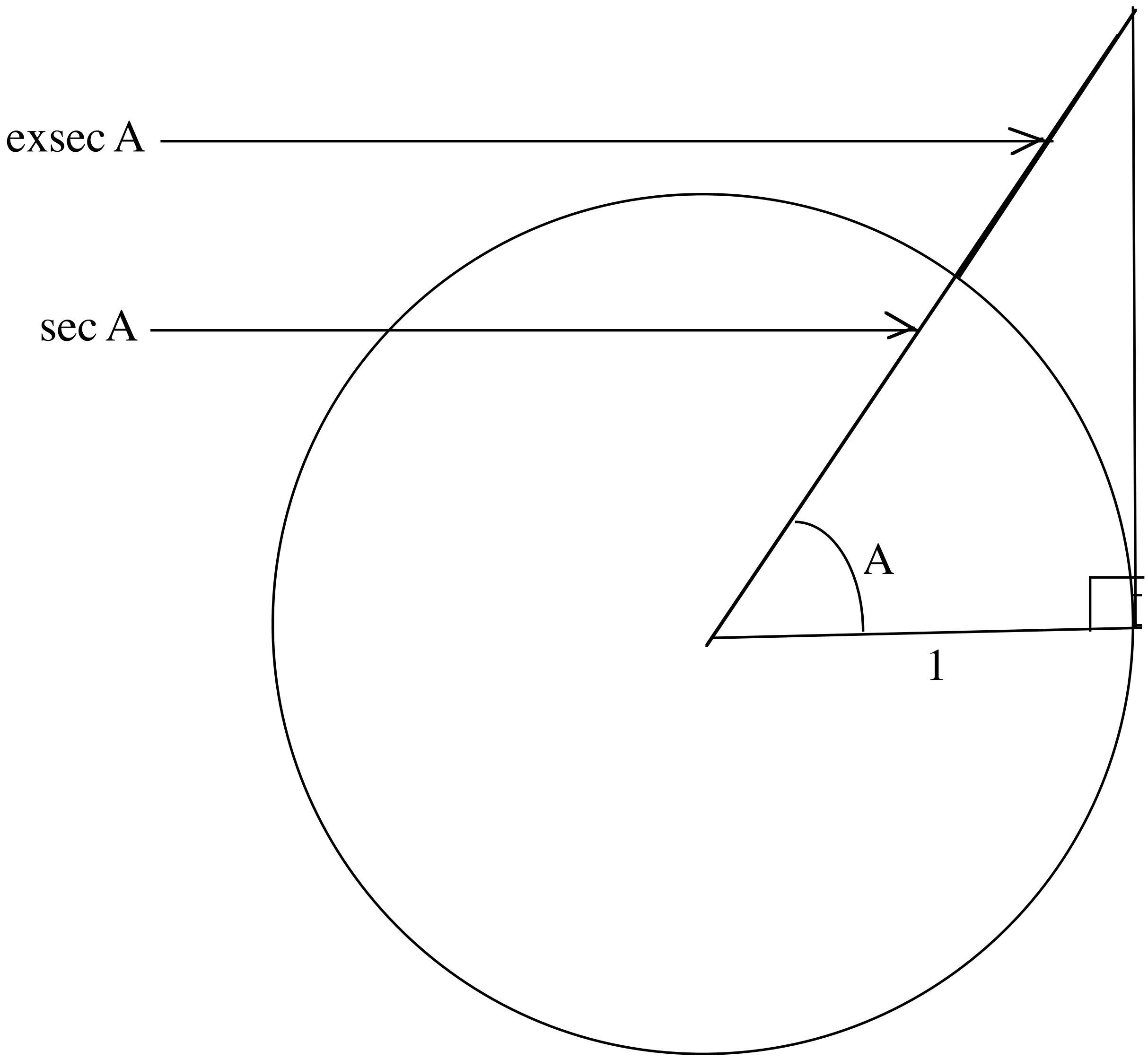

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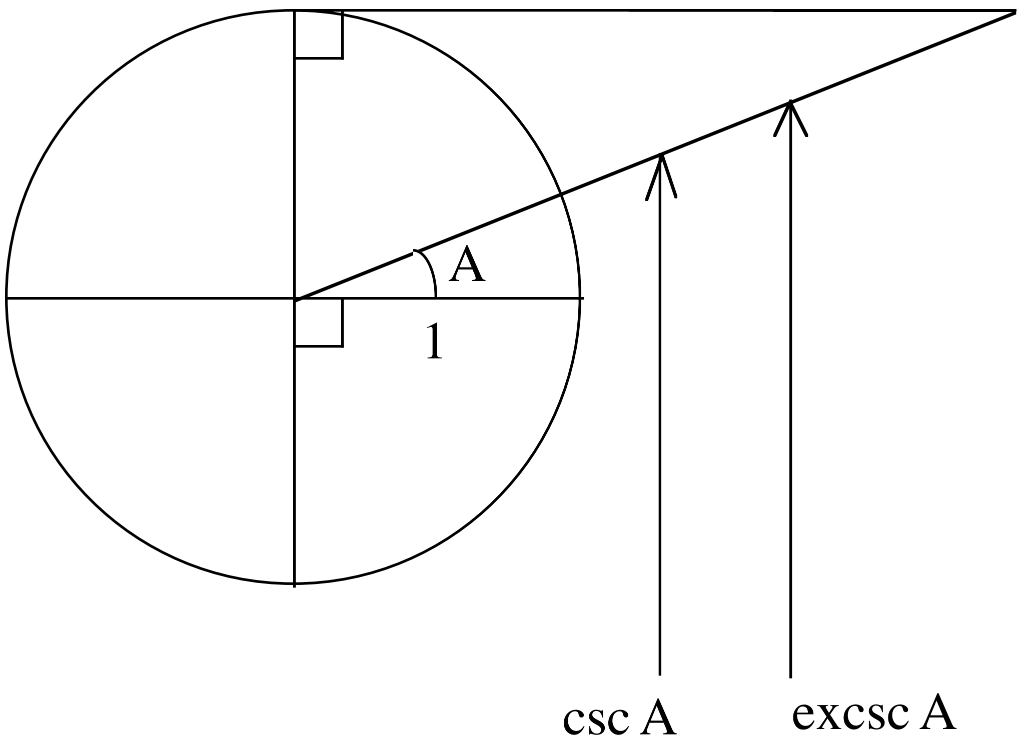

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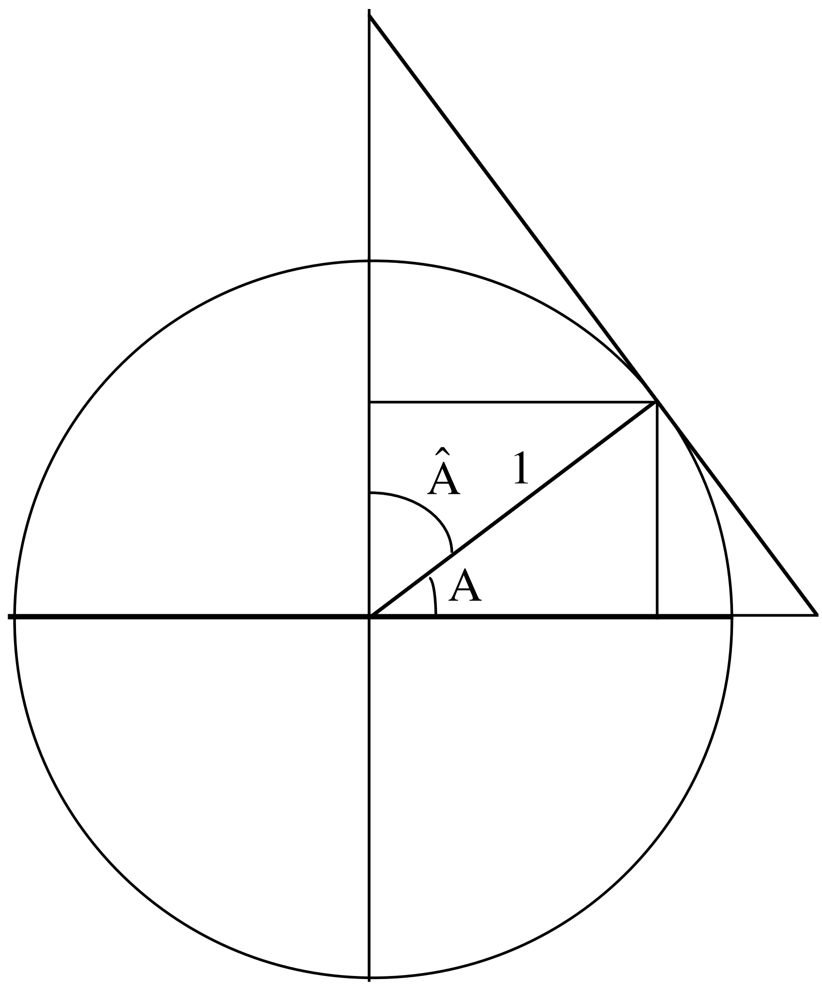

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identify the line segments

representing ie whose lengths are

$\sin A = \cos \hat{A}$

$\cos A = \sin \hat{A}$

$\tan A = \cot \hat{A}$

$\cot A = \tan \hat{A}$

$\sec A = \csc \hat{A}$

$\csc A = \sec \hat{A}$

$\text{vers}\, A = \text{covers}\, \hat{A}$

$\text{covers}\, A = \text{vers}\, \hat{A}$

$\text{exsec}\, A = \text{excsc}\, \hat{A}$

$\text{excsc}\, A = \text{exsec}\, \hat{A}$