Basic Notation & Terminology for Fields

#76 of Gottschalk's Gestalts

A Series Illustrating Innovative Forms
of the Organization & Exposition
of Mathematics
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Infinite Vistas Press
PVD RI
2002

GG76-1 (17)

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□ the basic notation & terminology
for a general abstract field
is taken over from that of
the three special concrete fields:
the field of rational numbers,
the field of real numbers,
the field of complex numbers;
ito operations
& not using any existential quantifiers,
a field consists of an underlying set
provided with

- two nullary operations
- two unary operations
- four binary operations

satisfying certain conditions = axioms;
allowing existential quantifiers in the definition
it is possible to define a field as a set
provided with
two binary operations
viz addition & multiplication
of a certain kind

□ the two basic nullary field operations
of a field F

Δ the additive identity element of F
$=_{dn}$ 0
$=_{rd}$ zero = oh
& ∴ the corresponding nullary operation in F is

$$\{\varnothing\} \to F$$
$$\varnothing \mapsto 0$$

Δ the multiplicative identity element of F
$=_{dn}$ 1
$=_{rd}$ one = unity
& ∴ the corresponding nullary operation in F is

$$\{\varnothing\} \to F$$
$$\varnothing \mapsto 1$$

□ the two basic unary field operations
of a field F

Δ negation in F
= the unary operation in F
$-: F \to F$
$a \mapsto -a$
wh
• $-$ $=_{rd}$ minus
• $-a$
$=_{rd}$ minus a
$=_{cl}$ the negation of a
• a $=_{cl}$ the negatee of $-a$

Δ reciprocation in F

= the partial unary operation in F

$^* : F_* \to F_*$

$a \mapsto a^*$

wh

- $^* =_{rd}$ recip
- a^*

$=_{rd}$ recip a

$=_{cl}$ the reciprocal of a

- $a =_{cl}$ the base of a^*

note: $a^* = a^{-1} = \frac{1}{a}$

it appears there is no recognized symbol

for the reciprocal of nonzero numbers in recent history,

probably because there was no special need for it;

the ancient Egyptians used unit fractions for fractions

and employed first an elongated oval

(hieroglyph for the open mouth)

and later a dot over a numeral to denote the reciprocal

□ the four basic binary field operations
of a field F

Δ addition in F
= the binary operation in F
$+: F \times F \rightarrow F$
$(a, b) \mapsto a + b$
wh
• $+$ $=_{rd}$ plus
• $a + b$
$=_{rd}$ a plus b
$=_{cl}$ the sum of a and b
• a $=_{cl}$ the first term / addend / summand of $a + b$
• b $=_{cl}$ the second term / addend / summand of $a + b$

Δ subtraction in F

$= $ the binary operation in F

$-\colon\ F \times F \ \to\ F$

$\quad (a, b)\ \mapsto\ a - b$

wh

• $-\ \ =_{rd}$ minus

• $a - b$

$=_{rd}$ a minus b

$=_{cl}$ the difference of a from b

• $a\ =_{cl}$ the first term of $a - b$

$\quad =$ the minuend of $a - b$

• $b\ =_{cl}$ the second term of $a - b$

$\quad =$ the subtrahend of $a - b$

Δ multiplication in F

= the binary operation in F

$\times \;=\; \cdot : F \times F \;\to\; F$

$(a, b) \;\mapsto\; a \times b \;=\; a \cdot b \;=\; ab$

wh

- $\times \;=\; \cdot \;=_{rd}$ times
- $a \times b \;=\; a \cdot b \;=\; ab$

$=_{rd}$ a times b

$=_{cl}$ the product of a and b

- a $=_{cl}$ the first term / factor / multiplier of $a \times b \;=\; a \cdot b \;=\; ab$
- b $=_{cl}$ the second term / factor / multiplier of $a \times b \;=\; a \cdot b \;=\; ab$

Δ division in F

= the partial binary operation in F

$\div \ = \ - \ = \ / : F \times F_* \to F$

$$(a, b) \mapsto a \div b = \frac{a}{b} = a / b$$

wh

- $\div \ =_{rd}$ divided by
- $- \ =_{rd}$ divided by = over
- $/ \ =_{rd}$ divided by = by

- $a \div b$

$=_{rd}$ a divided by b

$=_{cl}$ the quotient of a by b

- $\frac{a}{b}$

$=_{rd}$ a divided by b = a over b

= the fraction of a over b

with numerator a and denominator b

$=_{cl}$ the quotient of a by b

- a / b

$=_{rd}$ a divided by b = a by b

= the fraction of a over b

with numerator a and denominator b

$=_{cl}$ the quotient of a by b

- a $=_{cl}$ the first term / dividend of $a \div b = \frac{a}{b} = a / b$
- b $=_{cl}$ the second term / divisor of $a \div b = \frac{a}{b} = a / b$

□ the two basic numerical field operations
of a field F

Δ multiple - formation for F
= the function
$\mathbb{Z} \times F \to F$
$(n, a) \mapsto na$
wh
• na
$=_{rd}$ n times a
$=_{cl}$ the product of n and a
= the nth multiple of a
• n $=_{cl}$ the first term of na
= the numerical factor of na
= the multiplier of na
• a $=_{cl}$ the second term of na
= the field factor of na
= the multiplicand of na

Δ power - formation for F

= exponentiation for F

= the function

$F \times \mathbb{N} \cup F_* \times \overline{\mathbb{P}} \to F$

$(a, n) \mapsto a^n$

wh

• a^n

$=_{rd}$ a to the nth

$=_{cl}$ the power with base a and exponent n

= the nth power of a

• a $=_{cl}$ the base of a^n

• n $=_{cl}$ the exponent of a^n

note:

a^2 $=_{rd}$ a square(d) $=_{cl}$ the square of a

a^3 $=_{rd}$ a cube(d) $=_{cl}$ the cube of a

☐ word forms in the pattern

• noun
adjective
verb

• addition
additive
add

• subtraction
subtractive
subtract

• multiplication
multiplicative
multiply

• division
divisive
divide

☐ syntactic names of some symbols used for fields

- the addition sign $+$ = Greek cross
- the subtraction sign $-$ = horizontal bar
- the multiplication sign $\times$ = Saint Andrew's cross
- the multiplication sign $\cdot$ = mid dot
- the product ab of a and b = juxtaposition of a and b
- the division sign $\div$ = obelus
  (a combination of the horizonal bar and the colon)
- the division sign $-$ = the fraction sign $-$
  = horizontal bar

• the division sign / = the fraction sign / =

bend

bias

crossline

diagonal

oblique

scratch comma

separatrix

shilling

slant

slash

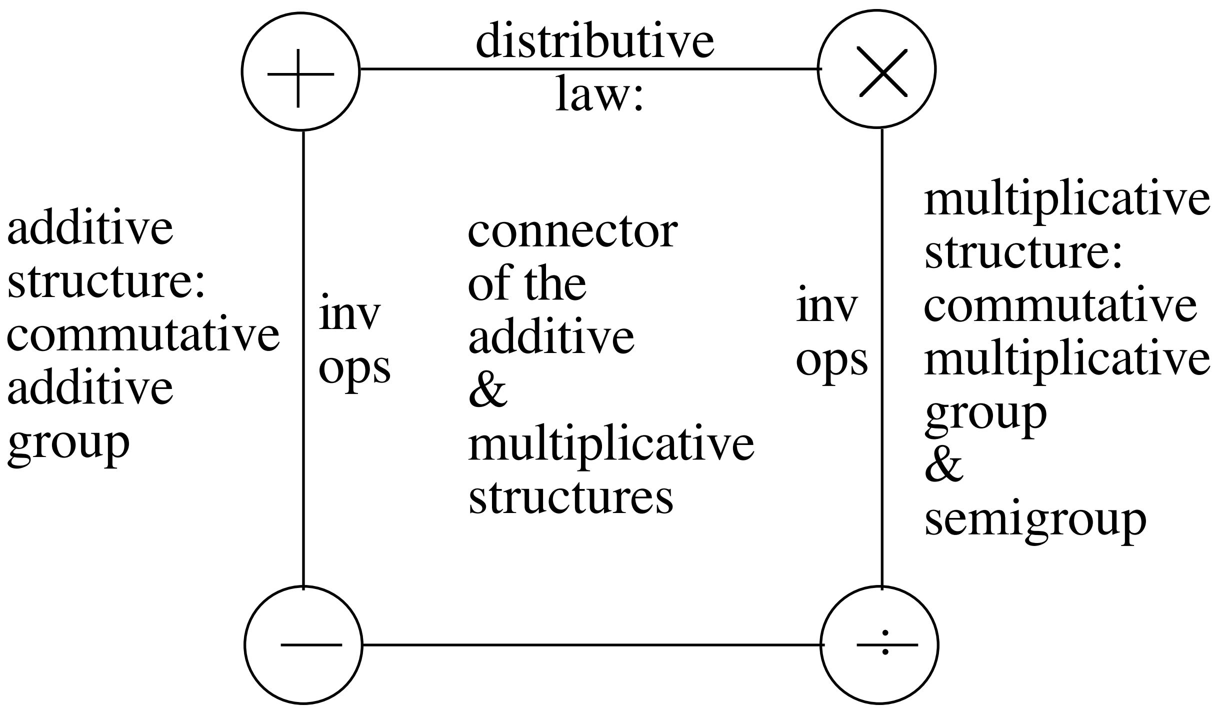
solidus

stroke

transverse

virgule

□ the structure square
of the four field basic binary operations



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