Thick & Thin Sets in Topological Spaces

#72 of Gottschalk's Gestalts

A Series Illustrating Innovative Forms
of the Organization & Exposition
of Mathematics
by Walter Gottschalk

Infinite Vistas Press
PVD RI
2002

500 Angell St #414
Providence RI 02906

GG72-2

D. dual thick / thin sets
in a top sp

let
• $X \in$ top sp
• $A \subset X$
then

(1) $A \in$ dense set

$= A \in$ dense

$=_{df} \overline{A} = X$

(1') $A \in$ border set

$= A \in$ border

$=_{df} \overset{o}{A} = \varnothing$

(2) $A \in$ wide set

$= A \in$ wide

$=_{df} A^{o-} = X$

(2') $A \in$ rare set

$= A \in$ rare

$=_{df} A^{-o} = \varnothing$

(3) A $\in$ full set

= A $\in$ full

$=_{df}$ A is the intersection of

a countable class of wide sets

= A is a countable intersection of wide sets

(3') A $\in$ meager set

= A $\in$ meager

$=_{df}$ A is the union of

a countable class of rare sets

= A is a countable union of rare sets

C. insights

• think of
dense / wide / full sets
as
big / large / pervasive / plentiful / thick sets

• think of
border / rare / meager sets
as
litttle / small / elusive / scant / thin sets

T. dual properties of thick / thin sets
in a top sp

let
• X $\in$ top sp
• A, B $\subset$ X
then

(1) A $\in$ dense $\Leftrightarrow$ A' $\in$ border
A $\in$ wide $\Leftrightarrow$ A' $\in$ rare
A $\in$ full $\Leftrightarrow$ A' $\in$ meager

(1') A $\in$ border $\Leftrightarrow$ A' $\in$ dense
A $\in$ rare $\Leftrightarrow$ A' $\in$ wide
A $\in$ meager $\Leftrightarrow$ A' $\in$ full

(2) every superset of a

dense

wide

full

set

has the same property

(2') every subset of a

border

rare

meager

set

has the same property

(3) $\overset{o}{A} \in$ wide $\Leftrightarrow$ $A \in$ wide

(3') $\overline{A} \in$ rare $\Leftrightarrow$ $A \in$ rare

(4) $A \in$ open

$\Rightarrow$

$A \in$ dense $\Leftrightarrow$ $A \in$ wide

(4') $A \in$ closed

$\Rightarrow$

$A \in$ border $\Leftrightarrow$ $A \in$ rare

(5) A and B are dense

&

A or B is open

$\Rightarrow$

$A \cap B$ is dense

(5') A and B are border

&

A or B is closed

$\Rightarrow$

$A \cup B$ is border

(6) any finite intersection of wide sets is wide

(6') any finite union of rare sets is rare

(7) any countable intersection of full sets is full

(7') any countable union of meager sets is meager

D. Baire spaces

• a Baire space
$=_{df}$ a topological space X st
every full subset of X is a dense subset of X
wiet the dual statement
every meager subset of X is a border subset of X

T. characterizations of Baire spaces

let

• X $\in$ top sp

then

tfsape

(1) X $\in$ Baire space

(2) every full subset of X is dense

(2') every meager subset of X is border

(3) every countable intersection
of wide subsets of X is dense

(3') every countable union
of rare subsets of X is border

(4) every countable intersection
of open dense subsets of X is dense

(4') every countable union
of closed border subsets of X is border

D. Baire' s property for sets
in a top sp

let
• $X \in$ top sp
• $A \subset X$
then

A has the property of Baire
= A has the Baire property
= A has Baire' s property
= A satisfies the condition of Baire
= A satisfies the Baire condition
= A satisfies Baire' s condition
$=_{df}$ there exists
an open subset B of X
and
a meager subset C of X
st
$A = B + C$

T. properties
of boundaries & border sets
in a top sp

let

- X $\in$ top sp
- A $\subset$ X

then

□ tfsape

- $\text{bdy}\, A = \varnothing$
- $\text{bdy}\, A' = \varnothing$
- $\overline{A} = \overset{o}{A}$
- $A'^{-} = A'^{o}$
- $A \in$ clopen set
- $A' \in$ clopen set

□ tfsape

- $\text{bdy}\, A = X$
- $\text{bdy}\, A' = X$
- $\overset{o}{A} = \varnothing \;\&\; \overline{A} = X$
- $A'^{o} = \varnothing \;\&\; A'^{-} = X$
- $A \in$ border set & $A \in$ dense set
- $A' \in$ border set & $A' \in$ dense set

☐ tfsape

- $A \subset \text{bdy}\, A$
- $\overset{o}{A} = \varnothing$
- $A \in$ border set

☐ tfsape

- $A \supset \text{bdy}\, A$
- $\overline{A} = A$
- $A \in$ closed set

☐ tfsape

- $A = \text{bdy}\, A$
- $\overline{A} = A$ & $\overset{o}{A} = \varnothing$
- $A \in$ closed border set

☐ tfsape

- $A' \subset \text{bdy}\, A$
- $\overline{A} = X$
- $A \in$ dense set

☐ tfsape

- $A' \supset \text{bdy}\, A$
- $\overset{o}{A} = A$
- $A \in$ open set

☐ tfsape

- $A' = \text{bdy}\, A$
- $\overline{A} = X$ & $\overset{o}{A} = A$
- $A \in$ open dense set

□ A $\in$ open or A $\in$ closed

$\Rightarrow$

int bdy A $=$ $\varnothing$

$\Leftrightarrow$

bdy A $\in$ border set