The Fascinating Fractious Egyptian Fractions

#69 of Gottschalk's Gestalts

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D. fractions

• a real fraction

$=_{df}$ an indicated quotient $\frac{a}{b}$

($=_{rd}$ a over b)

of real numbers a and b with $b \neq 0$

wh

the dividend a of the quotient

is called

the numerator of $\frac{a}{b}$

&

the divisor b of the quotient

is called

the denominator of $\frac{a}{b}$

&

either the numerator a or the denominator b

of $\frac{a}{b}$

is called

an ator of $\frac{a}{b}$

wh

ator $\leftarrow$ numer<u>ator</u> & denomin<u>ator</u>

&

the horizontal bar in the notation $\frac{a}{b}$

is called

the fraction bar of $\frac{a}{b}$;

note: the notion of real fraction

includes the notation of the fraction bar;

an alternative to the horizontal fraction bar $-$

is

the slant fraction bar / = the slash /

as in

$$\frac{a}{b} = a / b$$

• the value of a real fraction $\frac{a}{b}$

$=_{df}$ the real number

denoted by the real fraction $\frac{a}{b}$

• a simple real fraction

$=_{df}$ a real fraction st

neither ator contains a real fraction

• a complex real fraction

$=_{df}$ a real fraction st

at least one ator contains a real fraction

• a proper real fraction

$=_{df}$ a real fraction $\frac{a}{b}$ st

$b > 0$

&

$|a| < b$

wiet $\frac{|a|}{b} < 1$

• an improper real fraction

$=_{df}$ a real fraction $\frac{a}{b}$ st

$b > 0$

&

$|a| \geq b$

wiet $\frac{|a|}{b} \geq 1$

• a rational fraction

$=_{df}$ a real fraction $\frac{a}{b}$

whose ators a and b are both rational numbers

• a weakly common fraction

$=_{df}$ a real fraction $\frac{a}{b}$

whose ators a and b are both integers

• a (strictly) common fraction

$=_{df}$ a real fraction $\frac{a}{b}$

whose ators a and b are both positive integers

• a mixed fraction

$=_{df}$ an indicated sum of

a positive integer

&

a proper common fraction

wi denoted by juxtaposition

as

two and three - fourths $= 2 + \frac{3}{4} = 2\frac{3}{4}$

• an irreducible fraction
= a fraction in lowest term
$=_{df}$ a common fraction
whose ators are coprime

• a reducible fraction
$=_{df}$ a common fraction
whose ators are not coprime

• to reduce a common fraction
$=_{df}$ to divide both ators by
a plural common divisor of the ators
& thus obtain a new common fraction
that is equal (in value) to the original

• a unit fraction

$=_{df}$ a common fraction $\frac{1}{n}$ st

the numerator is unity

&

the denominator n is a plural integer;

thus the unit fractions are

$$\frac{1}{2},\ \frac{1}{3},\ \frac{1}{4},\ \frac{1}{5},\ \frac{1}{6},\ \ldots$$

• the unit fraction set

$=_{df}$ the set of all unit fractions

$$= \left\{\frac{1}{2},\ \frac{1}{3},\ \frac{1}{4},\ \frac{1}{5},\ \frac{1}{6},\ \ldots\right\}$$

• an egyptian fraction
$=_{df}$ an indicated nonzero finite sum of
distinct unit fractions
that are ordinarily written
in strictly decreasing order

• the value of an egyptian faction
$=_{df}$ the rational number that is its sum;
it is said that:
the egyptian fraction is a representation of the value,
the egyptian fraction represents the value,
the value is represented by the egyptian fraction

• a proper egyptian fraction
$=_{df}$ an egyptian fraction
whose value is strictly less than 1

• an improper egyptian fraction
$=_{df}$ an egyptian fraction
whose value is weakly greater than 1

• the length of an egyptian fraction
$=_{df}$ the number of terms
in the egyptian fraction

• an egyptian fraction
has minimum length
or
is of minimum length
or
is min - long
$=_{df}$ the length of the egyptian fraction
is minimal
among the lengths
of all the egyptian fractions
that have the same value
as the original egyptian fraction

• an egyptian fraction
has a minimum - maximum denominator
or
is min - max den
$=_{df}$ the maximum denominator
of the egyptian fraction
is minimal
among the maximum denominators
of all the egyptian fractions
that have the same value
as the original egyptian fraction

• an egyptian fraction is optimal
$=_{df}$ the egyptian fraction is
min - long
&
the maximum denominator
of the egyptian fraction
is minimal
among the maximum denominators
of all the min - long egyptian fractions
that have the same value
as the original egyptian fraction

☐ some convenient abbreviations

using the first three letters of the word

- integer = int
- positive = pos
- negative = neg
- numerator = num
- denominator = den
- maximum / maximal = max
- minimum / minimal = min
- optimum / optimal = opt

also

- nonnegative = nonneg
- nonpositive = nonpos

using the capitalized first letters of the words

- common fraction = CF
- egyptian fraction = EF
- unit fraction = UF

E. some proper EFs

- all unit fractions
- $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$
- $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$
- $\frac{1}{2} + \frac{1}{5} = \frac{7}{10}$
- $\frac{1}{2} + \frac{1}{3} + \frac{1}{7} = \frac{41}{42}$
- $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$
- $\frac{1}{3} + \frac{1}{5} + \frac{1}{7} = \frac{71}{105}$
- $\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10000} + \frac{1}{100000} = 0.11111$
- $\frac{1}{10} + \frac{1}{20} + \frac{1}{30} + \frac{1}{40} + \frac{1}{50} + \frac{1}{60} + \frac{1}{70} + \frac{1}{80} + \frac{1}{90} = \frac{7129}{25200}$
- $\frac{1}{8} + \frac{1}{120} = \frac{1}{9} + \frac{1}{45} = \frac{1}{10} + \frac{1}{30} = \frac{1}{12} + \frac{1}{20} = \frac{2}{15}$
- $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{35} + \frac{1}{276} + \frac{1}{2415} = \frac{18}{23}$

E. proper EFs wa

partial sums of geometric series

with a unit fraction as ratio

or equivalently

sums of geometric progressions

with a unit fraction as ratio

□ with ratio $= \frac{1}{2}$

- $\frac{1}{2} = \frac{1}{2}$
- $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$
- $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$
- $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{15}{16}$

etc

& ing

- $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^n} = \frac{2^n - 1}{2^n}$

wh $n \in \text{int} \geq 1$

□ with ratio $= \frac{1}{3}$

- $\frac{1}{3} = \frac{1}{3}$
- $\frac{1}{3} + \frac{1}{9} = \frac{4}{9}$
- $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} = \frac{13}{27}$
- $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} = \frac{40}{81}$

etc

& ing

- $\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots + \frac{1}{3^n} = \frac{\frac{1}{2}\left(3^n - 1\right)}{3^n}$

wh n $\in$ int ≥ 1

□ with ratio $= \frac{1}{r}$

wh r $\in$ int ≥ 2

- $\frac{1}{r} = \frac{1}{r}$
- $\frac{1}{r} + \frac{1}{r^2} = \frac{r+1}{r^2} = \frac{\frac{1}{r-1}\left(r^2 - 1\right)}{r^2}$
- $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} = \frac{r^2 + r + 1}{r^3} = \frac{\frac{1}{r-1}\left(r^3 - 1\right)}{r^3}$
- $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} = \frac{r^3 + r^2 + r + 1}{r^4} = \frac{\frac{1}{r-1}\left(r^4 - 1\right)}{r^4}$

etc

& ing

- $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \cdots + \frac{1}{r^n} = \frac{\frac{1}{r-1}\left(r^n - 1\right)}{r^n}$

wh n $\in$ int ≥ 1

- also termwise products of the above EFs by $\frac{1}{a}$

wh a $\in$ int ≥ 2

D. harmonic numbers

• the harnomic number of index n $\in$ nonneg int

= the nth harmonic number

$=_{dn}$ H_n wh H ← harmonic

$=_{df}$ $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{n}$

$= \sum_{i=1}^{n} \frac{1}{i}$

= the nth term of the harmonic sequence $(n \geq 1)$

= the nth partial sum of the harmonic series

• the harmonic sequence

$=_{df}$ $\left(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \cdots\right)$

wi convergent to 0

• the harmonic series

$=_{df}$ $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots$

wi divergent

R. the harmonic numbers H_n
wh $n \in$ nonneg int
are never integers
except for $H_0 = 0$ & $H_1 = 1$

R. note

- $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} = H_n - 1$

wh $n \in \text{int} \geq 2$

□ table of harmonic numbers H_n

from $n = 0$ to $n = 10$

- $H_0 = 0$
- $H_1 = 1$
- $H_2 = \frac{3}{2}$
- $H_3 = \frac{11}{6}$
- $H_4 = \frac{25}{12}$
- $H_5 = \frac{137}{60}$
- $H_6 = \frac{49}{20}$
- $H_7 = \frac{363}{140}$
- $H_8 = \frac{761}{280}$
- $H_9 = \frac{7129}{2520}$
- $H_{10} = \frac{7381}{2520}$

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E. some improper EFs

- $\frac{1}{2}+\frac{1}{3}+\frac{1}{6} = 1$
- $\frac{1}{2}+\frac{1}{3}+\frac{1}{5} = \frac{31}{30}$
- $\frac{1}{2}+\frac{1}{3}+\frac{1}{4} = H_4 - 1 = \frac{13}{12}$
- $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5} = H_5 - 1 = \frac{77}{60}$
- $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6} = H_6 - 1 = \frac{29}{20}$
- $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7} = H_7 - 1 = \frac{223}{140}$
- $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8} = H_8 - 1 = \frac{481}{280}$
- $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9} = H_9 - 1 = \frac{4609}{2520}$
- $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10} = H_{10} - 1 = \frac{4861}{2520}$

- $\frac{1}{2} + \frac{1}{3} + \frac{1}{15} + \frac{1}{22} + \frac{1}{35} + \frac{1}{63} + \frac{1}{99} = 1$
- $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{7} + \frac{1}{12} + \frac{1}{42} = \frac{3}{2}$
- $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{15} + \frac{1}{18} + \frac{1}{20} + \frac{1}{24} = 2$

E. representations of unity by EFs

with length ranging from 3 to 12;

for the given length the max den is least

- $1 = \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$
- $1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{12}$
- $1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{10} + \frac{1}{12} + \frac{1}{15}$
- $1 = \frac{1}{3} + \frac{1}{4} + \frac{1}{6} + \frac{1}{10} + \frac{1}{12} + \frac{1}{15}$
- $1 = \frac{1}{3} + \frac{1}{4} + \frac{1}{9} + \frac{1}{10} + \frac{1}{12} + \frac{1}{15} + \frac{1}{18}$
- $1 = \frac{1}{3} + \frac{1}{5} + \frac{1}{9} + \frac{1}{10} + \frac{1}{12} + \frac{1}{15} + \frac{1}{18} + \frac{1}{20}$
- $1 = \frac{1}{4} + \frac{1}{5} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{15} + \frac{1}{18} + \frac{1}{20} + \frac{1}{24}$
- $1 = \frac{1}{5} + \frac{1}{6} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{12} + \frac{1}{15} + \frac{1}{18} + \frac{1}{20} + \frac{1}{24}$
- $1 = \frac{1}{5} + \frac{1}{6} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{15} + \frac{1}{18} + \frac{1}{20} + \frac{1}{21} + \frac{1}{24} + \frac{1}{28}$
- $1 = \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10} + \frac{1}{14} + \frac{1}{15} + \frac{1}{18} + \frac{1}{20} + \frac{1}{24} + \frac{1}{28} + \frac{1}{30}$

E. representations of unity by EFs
with all dens odd

• with the fewest terms viz 9

$$1 = \frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}+\frac{1}{11}+\frac{1}{15}+\frac{1}{33}+\frac{1}{45}+\frac{1}{385}$$

• with the least max den viz 105

$$1 = \frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}+\frac{1}{11}+\frac{1}{33}+\frac{1}{35}+\frac{1}{45}+\frac{1}{55}+\frac{1}{77}+\frac{1}{105}$$

E. using

- $1 = \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$

repeatedly,

it follows that

- $\frac{3}{4}$

$= \frac{1}{2} + \frac{1}{4}$

$= \frac{1}{2} + \frac{1}{4} \times 1$

$= \frac{1}{2} + \frac{1}{8} + \frac{1}{12} + \frac{1}{24}$

$= \frac{1}{2} + \frac{1}{8} + \frac{1}{12} + \frac{1}{24} \times 1$

$= \frac{1}{2} + \frac{1}{8} + \frac{1}{12} + \frac{1}{48} + \frac{1}{72} + \frac{1}{144}$

etc

to an arbitrarily large number of terms

E. here are all 5 min - long representations of $\frac{19}{45}$ by EFs

but only one is optimal

- $\frac{19}{45}$

$$= \frac{1}{3} + \frac{1}{12} + \frac{1}{180} \in \text{min-long}$$

$$= \frac{1}{3} + \frac{1}{15} + \frac{1}{45} \in \text{min-long}$$

$$= \frac{1}{3} + \frac{1}{18} + \frac{1}{30} \in \text{min-long}$$

$$= \frac{1}{4} + \frac{1}{6} + \frac{1}{180} \in \text{min-long}$$

$$= \frac{1}{5} + \frac{1}{6} + \frac{1}{18} \in \text{opt}$$

R. an EF plus integer
that approximates
$\pi = 3.1415926\cdots$
is

$$3 + \frac{1}{13} + \frac{1}{17} + \frac{1}{173} = 3.1415269\cdots$$

• the Egyptian value of π
= the approximation to π
that the ancient Egyptians used

$$= \left(\frac{16}{9}\right)^2 = \frac{256}{81} = 3.16049\cdots$$

T. the greedy algorithm
to find EFs that represent proper CFs

let

- a, b $\in$ int st $1 < a < b$ & a and b are coprime
- n $=_{df}$ the least integer st $\frac{a}{b} > \frac{1}{n}$

then

- $\frac{a}{b} - \frac{1}{n} = \frac{an - b}{bn}$
- $a > an - b > 0$

T. rational numbers & egyptian fractions

• every positive rational number
is representable by
infinitely many EFs

• every rational number
is representable as
an integer plus an EF
in infinitely many ways

• every noninteger rational number
is representable as
an integer plus a proper EF
in infinitely many ways

- every positive rational number
is representable by
an EF
with length arbitrarily bounded below
&
with min den arbitrarily bounded below

- every positive rational number
is representable by
only finitely many EFs of a given length
if so representable at all

• every sum of n UFs
totaling strictly less than 1
is representable by
an EF of length n
wh n $\in$ pos int

• every proper CF

$$\frac{a}{b}$$

is representable by
an EF of length a

T. representations of 3 / n

by EFs of length 2

let

• n $\in$ int ≥ 4

then

tfsae:

• $\frac{3}{n}$ is representable by an EF of length 2

• n has a divisor that is congruent to 2 mod 3

ie

• $n = a(3k+2)$

for some pos int a

&

for some nonneg int k

R. note the following algebraic identity

in the real field say

• $$\frac{3}{a(3k+2)} = \frac{1}{a(k+1)} + \frac{1}{a(k+1)(3k+2)}$$

(dens $\neq 0$)

R. there are many simple algebraic identities
in the real field say
that are useful in the study of
UFs & EFs;
here are a few
(dens $\neq$ 0)

- $\dfrac{n}{ab} = \dfrac{1}{a\dfrac{a+b}{n}} + \dfrac{1}{b\dfrac{a+b}{n}}$

& the special cases

- $\dfrac{1}{ab} = \dfrac{1}{a(a+b)} + \dfrac{1}{b(a+b)}$
- $\dfrac{1}{a} = \dfrac{1}{a+1} + \dfrac{1}{a(a+1)}$

& $\therefore$

- $\dfrac{2}{a} = \dfrac{1}{a} + \dfrac{1}{a+1} + \dfrac{1}{a(a+1)}$

- $\dfrac{n}{a} = \dfrac{1}{x} + \dfrac{nx - a}{ax}$

& the special cases

- $\dfrac{1}{a} = \dfrac{1}{x} + \dfrac{x - a}{ax}$
- $\dfrac{2}{a} = \dfrac{1}{x} + \dfrac{2x - a}{ax}$
- $\dfrac{3}{a} = \dfrac{1}{x} + \dfrac{3x - a}{ax}$
- $\dfrac{2}{2n+1} = \dfrac{1}{n+1} + \dfrac{1}{(n+1)(2n+1)}$
- $\dfrac{3}{3n+2} = \dfrac{1}{n+1} + \dfrac{1}{(n+1)(3n+2)}$
- $\dfrac{4}{4n+3} = \dfrac{1}{n+1} + \dfrac{1}{(n+1)(4n+3)}$

etc

E. applications of the identity

- $\frac{1}{a} = \frac{1}{a+1} + \frac{1}{a(a+1)}$

include

- $\frac{1}{2} = \frac{1}{3} + \frac{1}{6}$
- $\frac{1}{3} = \frac{1}{4} + \frac{1}{12}$
- $\frac{1}{4} = \frac{1}{5} + \frac{1}{20}$
- $\frac{1}{5} = \frac{1}{6} + \frac{1}{30}$

etc

E. applications of the identity

- $\frac{2}{a} = \frac{1}{a} + \frac{1}{a+1} + \frac{1}{a(a+1)}$

include

- $\frac{2}{3} = \frac{1}{3} + \frac{1}{4} + \frac{1}{12}$
- $\frac{2}{5} = \frac{1}{5} + \frac{1}{6} + \frac{1}{130}$
- $\frac{2}{7} = \frac{1}{7} + \frac{1}{8} + \frac{1}{56}$
- $\frac{2}{9} = \frac{1}{9} + \frac{1}{10} + \frac{1}{90}$

etc

E. applications of the identity

- $\frac{1}{ab} = \frac{1}{a(a+b)} + \frac{1}{b(a+b)}$

include

- $\frac{1}{6} = \frac{1}{10} + \frac{1}{15}$
- $\frac{1}{12} = \frac{1}{21} + \frac{1}{28}$
- $\frac{1}{35} = \frac{1}{60} + \frac{1}{84}$
- $\frac{1}{63} = \frac{1}{112} + \frac{1}{144}$

E. applications of the identity

- $$\frac{n}{ab} = \frac{1}{a\frac{a+b}{n}} + \frac{1}{b\frac{a+b}{n}}$$

include

- $$\frac{2}{15} = \frac{1}{12} + \frac{1}{20}$$
- $$\frac{3}{14} = \frac{1}{6} + \frac{1}{21}$$
- $$\frac{4}{15} = \frac{1}{6} + \frac{1}{10}$$
- $$\frac{5}{21} = \frac{1}{6} + \frac{1}{14}$$

E. applications of the identity

- $$\frac{2}{2n+1} = \frac{1}{n+1} + \frac{1}{(n+1)(2n+1)}$$

include

- $$\frac{2}{3} = \frac{1}{2} + \frac{1}{6}$$
- $$\frac{2}{5} = \frac{1}{3} + \frac{1}{15}$$
- $$\frac{2}{7} = \frac{1}{4} + \frac{1}{28}$$
- $$\frac{2}{9} = \frac{1}{5} + \frac{1}{45}$$

etc

E. applications of the identity

- $\frac{3}{3n+2} = \frac{1}{n+1} + \frac{1}{(n+1)(3n+2)}$

include

- $\frac{3}{5} = \frac{1}{2} + \frac{1}{10}$
- $\frac{3}{8} = \frac{1}{3} + \frac{1}{24}$
- $\frac{3}{11} = \frac{1}{4} + \frac{1}{44}$
- $\frac{3}{14} = \frac{1}{5} + \frac{1}{70}$

etc

E. applications of the identity

- $\frac{1}{a} = \frac{1}{x} + \frac{x-a}{ax}$

include

- $\frac{1}{2} = \frac{1}{3} + \frac{1}{6}$
- $\frac{1}{3} = \frac{1}{4} + \frac{1}{12}$
- $\frac{1}{4} = \frac{1}{6} + \frac{1}{12}$
- $\frac{1}{5} = \frac{1}{9} + \frac{2}{35}$ $(\neg$ EF)

E. applications of the identity

- $\frac{2}{a} = \frac{1}{x} + \frac{2x - a}{ax}$

include

- $\frac{2}{3} = \frac{1}{2} + \frac{1}{6}$
- $\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$ (¬EF)
- $\frac{2}{3} = \frac{1}{4} + \frac{5}{12}$ (¬EF)
- $\frac{2}{3} = \frac{1}{5} + \frac{7}{15}$ (¬ EF)

E. applications of the identity

- $\frac{3}{a} = \frac{1}{x} + \frac{3x - a}{ax}$

include

- $\frac{3}{25} = \frac{1}{10} + \frac{1}{50}$
- $\frac{3}{55} = \frac{1}{20} + \frac{1}{220} = \frac{1}{22} + \frac{1}{110}$
- $\frac{3}{121} = \frac{1}{44} + \frac{1}{484}$
- $\frac{3}{149} = \frac{1}{50} + \frac{1}{7450}$

N. a convenient on - the - line notation
to denote
sums of UFs
is the following:

- $\frac{1}{a} =_{dn} [a]$
- $\frac{1}{a} + \frac{1}{b} =_{dn} [a, b]$
- $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} =_{dn} [a, b, c]$
- $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} =_{dn} [a, b, c, d]$

etc
wh
$[\cdots] =_{rd}$ brac $\cdots$
&
brac $\leftarrow$ <u>brac</u>ket

E. some proper CFs
represented by
all min - long EFs

• den = 2

$$\frac{1}{2} = [2]$$

• den = 3

$$\frac{1}{3} = [3]$$

$$\frac{2}{3} = [2,\ 6]$$

• den = 4

$$\frac{1}{4} = [4]$$

$$\frac{2}{4} = \frac{1}{2} = [2]$$

$$\frac{3}{4} = [2,\ 4]$$

- den $= 5$

$\frac{1}{5} = [5]$

$\frac{2}{5} = [3, 15]$

$\frac{3}{5} = [2, 10]$

$\frac{4}{5} = [2, 4, 20] = [2, 5, 10]$

- den $= 6$

$\frac{1}{6} = [6]$

$\frac{2}{6} = \frac{1}{3} = [3]$

$\frac{3}{6} = \frac{1}{2} = [2]$

$\frac{4}{6} = \frac{2}{3} = [2, 6]$

$\frac{5}{6} = [2, 3]$

• den = 7

$\frac{1}{7} = [7]$

$\frac{2}{7} = [4,\ 28]$

$\frac{3}{7} = [3,\ 11,\ 231] = [3,\ 12,\ 84] = [3,\ 14,\ 42]$

$= [3,\ 15,\ 35] = [4,\ 6,\ 84] = [4,\ 7,\ 28]$

$\frac{4}{7} = [2,\ 14]$

$\frac{5}{7} = [2,\ 5,\ 70] = [2,\ 6,\ 21] = [2,\ 7,\ 14]$

$\frac{6}{7} = [2,\ 3,\ 42]$

- den = 8

$\frac{1}{8} = [8]$

$\frac{2}{8} = \frac{1}{4} = [4]$

$\frac{3}{8} = [3, 24] = [4, 8]$

$\frac{4}{8} = \frac{1}{2} = [2]$

$\frac{5}{8} = [2, 8]$

$\frac{6}{8} = \frac{3}{4} = [2, 4]$

$\frac{7}{8} = [2, 3, 24] = [2, 4, 8]$

- den = 9

$\frac{1}{9} = [9]$

$\frac{2}{9} = [5, 45] = [6, 18]$

$\frac{3}{9} = \frac{1}{3} = [3]$

$\frac{4}{9} = [3, 9]$

$\frac{5}{9} = [2, 18]$

$\frac{6}{9} = \frac{2}{3} = [2, 6]$

$\frac{7}{9} = [2, 4, 36] = [2, 6, 9]$

$\frac{8}{9} = [2, 3, 18]$

• den = 10

$$\frac{1}{10} = [10]$$

$$\frac{2}{10} = \frac{1}{5} = [5]$$

$$\frac{3}{10} = [4,\ 20] = [5,\ 10]$$

$$\frac{4}{10} = \frac{2}{5} = [3,\ 15]$$

$$\frac{5}{10} = \frac{1}{2} = [2]$$

$$\frac{6}{10} = \frac{3}{5} = [2,\ 10]$$

$$\frac{7}{10} = [2,\ 5]$$

$$\frac{8}{10} = \frac{4}{5} = [2,\ 4,\ 20] = [2,\ 5,\ 10]$$

$$\frac{9}{10} = [2,\ 3,\ 15]$$

• den = 11

$\frac{1}{11}$ = [11]

$\frac{2}{11}$ = [6, 66]

$\frac{3}{11}$ = [4, 44]

$\frac{4}{11}$ = [3, 33]

$\frac{5}{11}$ = [3, 9, 99] = [3, 11, 33] = [4, 5, 220]

$\frac{6}{11}$ = [2, 22]

$\frac{7}{11}$ = [2, 8, 88] = [2, 11, 22]

$$\frac{8}{11} = [2,\ 5,\ 37,\ 4070] = [2,\ 5,\ 38,\ 1045]$$
$$= [2,\ 5,\ 40,\ 440] = [2,\ 5,\ 44,\ 220]$$
$$= [2,\ 5,\ 45,\ 198] = [2,\ 5,\ 55,\ 110]$$
$$= [2,\ 5,\ 70,\ 77] = [2,\ 6,\ 17,\ 561]$$
$$= [2,\ 6,\ 18,\ 198] = [2,\ 6,\ 21,\ 77]$$
$$= [2,\ 6,\ 22,\ 66] = [2,\ 7,\ 12,\ 924]$$
$$= [2,\ 7,\ 14,\ 77] = [2,\ 8,\ 10,\ 440]$$
$$= [2,\ 8,\ 11,\ 88]$$
$$\frac{9}{11} = [2,\ 4,\ 15,\ 660] = [2,\ 4,\ 16,\ 176]$$
$$= [2,\ 4,\ 20,\ 55] = [2,\ 4,\ 22,\ 44]$$
$$= [2,\ 5,\ 10,\ 55]$$
$$\frac{10}{11} = [2,\ 3,\ 14,\ 231] = [2,\ 3,\ 15,\ 110]$$
$$= [2,\ 3,\ 22,\ 33]$$

• den = 12

$$\frac{1}{12} = [12]$$

$$\frac{2}{12} = \frac{1}{6} = [6]$$

$$\frac{3}{12} = \frac{1}{4} = [4]$$

$$\frac{4}{12} = \frac{1}{3} = [3]$$

$$\frac{5}{12} = [3,\ 12] = [4,\ 6]$$

$$\frac{6}{12} = \frac{1}{2} = [2]$$

$$\frac{7}{12} = [2,\ 12] = [3,\ 4]$$

$$\frac{8}{12} = \frac{2}{3} = [2,\ 6]$$

$$\frac{9}{12} = \frac{3}{4} = [2,\ 4]$$

$$\frac{10}{12} = \frac{5}{6} = [2,\ 3]$$

$$\frac{11}{12} = [2,\ 3, 12] = [2,\ 4,\ 6]$$

□ comments

• there are many algorithms
for the conversion of common fractions
into egyptian fractions

• there are many
unsolved problems
& unanswered questions
& challenging conjectures
about
unit fractions
& egyptian fractions
& related diophantine equations

• a study of the above topics
often requires a computer,
the more powerful the better

C. some other kinds of fractions are:

- complex number fractions
- continued fractions
- Farey fractions
- partial fractions
- percentages
- elements of the quotient field of an integral domain
- fractions in a field
- n - ary fractions ($n \in int \geq 2$)

inp

- binary fractions
- decimal fractions
- duodecimal fractions
- hexadecimal fractions