Repunits

#67 of Gottschalk's Gestalts

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D. repunits

let

• n $\in$ pos int

then

• the repunit for / of / with index n

= the nth repunit

$=_{dn}$ R_n = R(n) = Rn

$=_{df}$ $\dfrac{10^n - 1}{10 - 1} = \dfrac{10^n - 1}{9}$

= the number whose base 10 numeral
consists of n consecutive unit digits

= n one' s

= n 1' s

= $111 \cdots 111$ (n digits)

& $\therefore$

$R_1 = 1$

$R_2 = 11$

$R_3 = 111$

$R_4 = 1111$

$R_5 = 11111$

$R_6 = 111111$

etc

N. origin of notation

- repunit ← rep eated unit

- R_n = R(n) ← repunit of index n

☐ prime factorizations of repunits $R(n)$ for $1 \le n \le 16$

- $R(1) = 1$ (pf)
- $R(2) = 11$ (pf)
- $R(3) = 3 \times 37$ (pf)
- $R(4) = 11 \times 101$ (pf)
- $R(5) = 41 \times 271$ (pf)
- $R(6) = 3 \times 7 \times 11 \times 13 \times 37$ (pf)
- $R(7) = 239 \times 4649$ (pf)
- $R(8) = 11 \times 73 \times 101 \times 137$ (pf)
- $R(9) = 3^2 \times 37 \times 333667$ (pf)
- $R(10) = 11 \times 41 \times 271 \times 9091$ (pf)
- $R(11) = 21649 \times 513239$ (pf)
- $R(12) = 3 \times 7 \times 11 \times 13 \times 37 \times 101 \times 9901$ (pf)
- $R(13) = 53 \times 79 \times 265371653$ (pf)
- $R(14) = 11 \times 239 \times 4649 \times 909091$ (pf)
- $R(15) = 3 \times 31 \times 37 \times 41 \times 271 \times 2906161$ (pf)
- $R(16) = 11 \times 17 \times 73 \times 101 \times 137 \times 5882352$ (pf)

- the complete prime factorizations of the repunits $R(n)$ for $1 \le n < 236$ are known (2001) with 4 exceptions: $n = 197, 223, 227, 233$

R. a systematic ' algebraic type' factoring of R(n)
for composite plural integers n
may be illustrated by
the following example for n = 12
whose plural proper factors are 2, 3, 4, 6:
R(12)
$= 11 \times 10101010101$
$= 111 \times 1001001001$
$= 1111 \times 100010001$
$= 111111 \times 1000001$

T. let

- m, n $\in$ pos int

then

- m divides n $\Rightarrow$ R(m) divides R(n)

- R(m) & R(n) divide R(mn)

- R(m) and R(n) are coprime $\Leftrightarrow$ m and n are coprime

- n is composite $\Rightarrow$ R(n) is composite;
  the converse fails

- R(n) is prime $\Rightarrow$ n is prime;
  the converse fails

R. divisibility properties of repunits

• every repunit whose index is divisible by 2
has 11 as a prime factor

• every repunit whose index is divisible by 3
has 3 & 37 as prime factors

• every repunit whose index is divisible by 4
has 11 & 101 as prime factors

• every repunit whose index is divisible by 5
has 41 & 271 as prime factors

• every repunit whose index is divisible by 6
has 3 & 7 & 11 & 13 & 37 as prime factors

etc

☐ the known prime repunits

- $R(2)$ = the 1st prime repunit
- $R(19)$ = the 2nd prime repunit
- $R(23)$ = the 3rd prime repunit
- $R(317)$ = the 4th prime repunit
- $R(1031)$ = the 5th prime repunit
  & the largest known (2001) prime repunit
- there are no other prime repunits $R(n)$
for $1 \leq n \leq 45{,}000$
- R(49081) is a probable prime
- R(86453) is a probable prime
- it is not known (2001) whether
there are infinitely many prime repunits

C. repunits to base 10
may be generalized to
repunits to base b
where b is any plural integer;
the repunit
of positive integer index n to base b

$$=_{df} \quad \frac{b^n - 1}{b - 1} \ ;$$

note that
the repunit of index n to base 2
is the Mersenne number

$$M_n = 2^n - 1$$