The Hyperpower Function

#66 of Gottschalk's Gestalts

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D, &T. the hyperpower function

- the hyperpower function

= the infinitely stacked / storied exponential function

$y = p(x) =_{df} \lim_{n\to\infty} p_n(x)$ iie

wh

$x, y \in$ pos real nr var

$n \in$ pos int var

$p_1(x) =_{df} x \quad (x > 0)$

$p_{n+1}(x) =_{df} x^{p_n(x)} \quad (x > 0) \quad$ (pv)

& ∴

$p_1(x) = x$

$p_2(x) = x^x$

$p_3(x) = x^{x^x}$

etc

all $p_n(x)$ $(n \in \mathbb{P})$ are defined uniquely

& are positive for $x > 0$

• seq $\left(p_n(x) \mid n \in \mathbb{P}\right)$

converges for $x \in I = \mathbb{R}\left[\frac{1}{e^e}, e^{\frac{1}{e}}\right]$

&

diverges for $x \in \mathbb{R}_+ - I$

• the domain of $y = p(x)$

$= \text{dmn } p$

$= \mathbb{R}\left[\frac{1}{e^e}, e^{\frac{1}{e}}\right]$

• the range of $y = p(x)$

$= \text{rng } p$

$= \mathbb{R}\left[\frac{1}{e}, e\right]$

• $p\left(\frac{1}{e^e}\right) = \frac{1}{e}$

• $p(1) = 1$

• $p(\sqrt{2}) = 2$

• $p\left(e^{\frac{1}{e}}\right) = p(\sqrt[e]{e}) = e$

• $p\left(a^{\frac{1}{a}}\right) = p(\sqrt[a]{a})$ if $a \in$ real no st $\frac{1}{e} \leq a \leq e^{\frac{1}{e}}$

• $p(x)$ is strictly increasing on its domain $\frac{1}{e^e} \leq x \leq e^{\frac{1}{e}}$

• the graph of $y = p(x)$
has a single point of inflection
wi near (0.3944, 0.5819);
to the left the graph is convex up;
to the right the graph is convex down

- $e = 2.71828\ 18284\cdots$

- $e^e = 15.15426\ 22414\cdots$

- $e^{\frac{1}{e}} = \sqrt[e]{e} = 1.44466\ 78610\cdots$

- $\frac{1}{e} = 0.36787\ 94412\cdots$

- $\frac{1}{e^e} = 0.0659880\cdots$

- $\frac{1}{e^{\frac{1}{e}}} = \frac{1}{\sqrt[e]{e}} = 0.69220\ 06276\cdots$