Intro Info on Regular Polygons

#65 of Gottschalk's Gestalts

A Series Illustrating Innovative Forms
of the Organization & Exposition
of Mathematics
by Walter Gottschalk

Infinite Vistas Press
PVD RI
2001

GG65-1 (61)

500 Angell St #414
Providence RI 02906

GG65-2

□ all considerations take place
in the euclidean plane

D. a polygon
$=_{df}$ an ordered pair (V, P)
where
V is a cyclic sequence
of n distinct points of the euclidean plane
with n an integer at least 3
&
P is the subset of the euclidean plane
which is
the union of the n line segments
connecting the n adjacent point pairs of V,

V being called
the vertex cycle of the polygon (V, P)
&
the n points of V being called
the vertices of the polygon (V, P)
&
P being called
the perimeter of the polygon (V, P)
&
the n line segments connecting
the adjacent point pairs of V being called
the sides of the polygon (V, P)
&
n being called
the vertex - number / side - number / order
of the polygon (V, P)

note: altho P is uniquely determined by V
in a polygon (V,P),
in most cases considered P uniquely determines V
& P is thought of as the polygon

D. an n - gon

wh n $\in$ int $\geq$ 3

$=_{df}$ a polygon with exactly n vertices

= an n - vertex polygon

= a polygon with exactly n sides

= an n - sided polygon

D. diagonals of polygons

• a diagonal of a polygon
$=_{df}$ a line segment joining
two nonadjacent vertices of the polygon

• a k - diagonal of an n - gon
wh $n, k \in$ int st
$n \geq 3$ & $2 \leq k \leq n - 2$
$=_{df}$ a diagonal of the n - gon
whose endpoints have
$k - 1$ consecutive vertices of the n - gon
strictly inbetween them
or equivalently
which spans k consecutive sides of the polygon

D. basic properties of polygons

let

• (V, P) $\in$ polygon

then

• (V, P) $\in$ simple

$=_{df}$ each pair of adjacent sides

intersects exactly in their common endpoint

&

each pair of nonadjacent sides

is disjoint

= P $\in$ simple closed curve

• (V, P) $\in$ convex

$=_{df}$ (V, P) $\in$ simple & int P $\in$ convex

• (V, P) $\in$ concave

$=_{df}$ (V, P) $\in$ simple & int P $\notin$ convex

• $(V, P) \in$ proper

$=_{df}$ no two adjacent sides of (V, P) are collinear

$=$ no three consecutive vertices of (V, P) are collinear

• $(V, P) \in$ prosimple

$=_{df}$ $(V, P) \in$ proper & simple

wh

prosimple $\leftarrow$ pro<u></u>per + simple

• $(V, P) \in$ proconvex

$=_{df}$ $(V, P) \in$ proper & convex

wh

proconvex $\leftarrow$ proper + convex

• $(V, P) \in$ proconcave

$=_{df}$ $(V, P) \in$ proper & concave

wh

proconcave $\leftarrow$ proper + concave

D. the area of a simple polygon (V, P)
$=_{df}$ the area of int P

C. we may think of
a proconvex n - gon ($n \in$ int ≥ 3)
variously and equivalently
in three ways as:

- the 0 - dimensional set of its n vertices
- the 1 - dimensional union of its n sides
- the 2 - dimensional convex hull

of either of the above sets,
the hull being described as
a closed polygonal plate

D. associated angles of simple polygons

let

• $(V,P) \in$ simple polygon

• $A \in$ vertex of (V,P)

then

• the interior angle of (V,P) at A

= the angle of (V,P) at A

$=_{df}$ the sectorial angle

with vertex A

whose sides are the extended sides

of the polygon (V,P)

issuing from A,

both proximally,

&

which intersects $\operatorname{int} P$ locally at A;

a simple n - gon ($n \in \text{int} \geq 3$)

has exactly n interior angles,

one at each vertex

• the external angle of (V, P) at A
$=_{df}$ the sectorial angle
with vertex A
whose sides are the extended sides
of the polygon (V, P)
issuing from A,
both proximally,
&
which does not intersect int P locally at A;
a simple n - gon ($n \in$ int ≥ 3)
has exactly n external angles,
one at each vertex

D. exterior angles of a simple convex polygon

let

• (V, P) $\in$ simple convex polygon

• A $\in$ vertex of (V, P)

then

• an exterior angle of (V, P) at A

$=_{df}$ one of the two sectorial angles

with vertex A

whose sides are the extended sides

of the polygon (V, P)

issuing from A,

one proximally and one distally,

&

which do not intersect int P locally at A;

a simple convex n - gon ($n \in int \geq 3$)

has exactly 2n exterior angles,

two at each vertex

D. properties of angles of simple polygons

let

• $(V, P) \in$ simple polygon

then

• an interior angle of (V, P)

$=_{df}$ salient or flat or re - entrant

according as

the interior angle is

strictly less than

or

equal to

or

strictly greater than

a straight angle of 180^{o}

N. frequently useful notation

for the cycliclly ordered vertices of a polygon

□ capital English letters in alphabetic order

- A,B,C for a triangle
- A,B,C,D for a quadrilateral
- A,B,C,D,E for a pentagon

etc

□ A with consecutive integer subscripts starting with 1

- $A_1, A_2, A_3, \cdots, A_n$ for an n - gon

wh n $\in$ int ≥ 3

D. kinds of polygons

• an equilateral polygon

$=_{df}$ a polygon st

all sides of the polygon

are equal

ie

have the same length

• an equiangular polygon

$=_{df}$ a simple polygon st

all interior angles of the polygon

are equal

ie

have the same measure

• a regular polygon

$=_{df}$ an equilateral equiangular polygon

• a cyclic polygon

$=_{df}$ a polygon whose vertices

all lie on a necessarily unique circle

called

the circumscribed circle of the polygon

or more briefly

the circumcircle of the polygon,

the center of the circumcircle

being called

the circumcenter of the polygon

or more briefly

the center of the polygon

&

a / the radius of the circumcircle

being called

a / the circumradius of the polygon

or more briefly

a / the radius of the polygon

&

a / the diameter of the circumcircle

being called

a / the circumdiameter of the polygon

• a gyric polygon

$=_{df}$ a convex polygon st

all sides are internally tangent to

a necessarily unique circle

called

the inscribed circle of the polygon

or more briefly

the incircle of the polygon,

the center of the incircle

being called

the incenter of the polygon

&

an / the radius of the incircle being called

an / the inradius of the polygon

&

an / the diameter of the incircle being called

an / the indiameter of the polygon

R. the regular polygons
are
equilateral
equiangular
cyclic
gyric
and
the most symmetric of all polygons;
for a given $n \in$ int ≥ 3
all regular n - gons are similar
and $\therefore$
one may sometimes speak of
<u>the</u> regular n – gon

D. a symmetry of a geometric object
$=_{df}$ a one - to - one transformation of the object onto itself
that is isometric ie distance - preserving;
thus the notion of a symmetry of a regular polygon
is defined uniquely whether we think of the polygon as
the corners or the frame or the plate

R. symmetries for a regular polygon
• the center of rotational symmetry of a regular polygon
= the center = the circumcenter = the incenter
wi also the center of reflective symmetry
if the order is even
• the n axes of reflective symmetry
of a regular n - gon ($n \in$ int ≥ 3) are:
for odd n, the n circumdiameters thru a vertex
which coincide with
the n perpendicular bisectors of the sides;
for even n, the n / 2 circumdiameters
thru opposite vertex pairs
and the n / 2 perpendicular bisectors
of pairs of opposite parallel sides;

D. dihedral groups

• the nth dihedral group wh $n \in$ int ≥ 3

$=_{dn}$ Δ_n

$=_{df}$ the group of symmetries

of the regular polygon P_n

= the symmetry group of P_n

= the group of order 2n generated by

the rotation R of P_n thru $360^o / n$

about the center of P_n

and

the reflection F of P_n in a line

thru a vertex of P and the center of P_n

viz

$1, R, R^2, \cdots, R^{n-1}$,

$F, RF, R^2F, \cdots, R^{n-1}F$

= the group of order 2n consisting of

the n rotational congruences & the n reflective congruences;

a cyclic subgroup C_n of Δ_n of order n

consists of the n rotational congruences $1, R, R^2, \cdots, R^{n-1}$

D. notions for a regular polygon

• a spoke of the polygon

$=_{df}$ a line segment joining

the center of the polygon & a vertex of the polygon

= a circumradius of the polygon to a vertex of the polygon;

a regular n - gon ($n \in$ int ≥ 3)

has exactly n spokes,

one to a vertex

• a central triangle of the polygon

$=_{df}$ a triangle whose sides are

a side of the polygon & the two spokes to its endpoints;

a regular n - gon ($n \in$ int ≥ 3)

has exactly n central triangles,

one to a side of the polygon

N. notation for a regular polygon P_n of order n
wh n $\in$ int ≥ 3

- n = the number of the vertices of the polygon
  = the vertex - number of the polygon
  = the number of the sides of the polygon
  = the side - number of the polygon
  = the number of
  the central / interior / external angles
  of the polygon
  = the angle - number of the polygon

- a = the length of a side of the polygon
  = the side - length of the polygon
  = the side of the polygon

- d_k wh $k \in$ int st $2 \le k \le n - 2$

= the length of a diagonal of the polygon
spanning k consecutive sides of the polygon

= the k - diagonal - length of the polygon

= the k - diagonal of the polygon

- p = the perimeter of the polygon
  = the sum of the lengths of all sides of the polygon

O = the center of the circumcircle of the polygon
= the circumcenter of the polygon
= the center of the incircle of the polygon
= the incenter of the polygon
= the center of the polygon

• R = the radius of the circumcircle of the polygon
= the circumradius of the polygon
= the radius of the polygon

• r = the radius of the incircle of the polygon
= the inradius of the polygon
= the apothem of the polygon

• $C(O, R)$ = the circle with center O and radius R
= the circle passing thru all vertices of the polygon
= the circumscribing circle of the polygon
= the circumcircle of the polygon

• $C(O, r)$ = the circle with center O and radius r
= the circle tangent to all sides of the polygon
= the inscribed circle of the polygon
= the incircle of the polygon

- α = the interior angle of the polygon

- β = the exterior angle of the polygon

- γ = the external angle of the polygon

- δ = the base angle of the polygon
  = the base angle of a central triangle of the polygon

- ϑ = the central angle of the polygon
  = the apex angle of a central triangle of the polygon
  = the sectorial angle
  with vertex at the center of the polygon
  and subtended by a side of the polygon
  = the sectorial angle
  between two consecutive spokes of the polygon
  that is less than a straight angle

☐ formulary for the regular n - sided polygon P_n

wh $n \in$ int ≥ 3

- $a = 2\tan\frac{\vartheta}{2}\, r = 2\sin\frac{\vartheta}{2}\, R = 2\sqrt{R^2 - r^2}$

- $r = \frac{1}{2}\cot\frac{\vartheta}{2}\, a = \cos\frac{\vartheta}{2}\, R = \frac{1}{2}\sqrt{4R^2 - a^2}$

- $R = \frac{1}{2}\csc\frac{\vartheta}{2}\, a = \sec\frac{\vartheta}{2}\, r = \frac{1}{2}\sqrt{a^2 + 4r^2}$

- $a^2 + 4r^2 = 4R^2$

- $p = na = 2n\tan\frac{\vartheta}{2}\, r = 2n\sin\frac{\vartheta}{2}\, R = 2n\sqrt{R^2 - r^2}$

- $d_k = d_{n-k}$ wh $k \in$ int st $2 \le k \le n-2$

$$= \sin k\frac{\vartheta}{2}\csc\frac{\vartheta}{2}\ a$$

$$= 2\sin k\frac{\vartheta}{2}\sec\frac{\vartheta}{2}\ r$$

$$= 2\sin k\frac{\vartheta}{2}\ R$$

- the number of the diagonals of P_n

$$= \frac{1}{2}n(n-3)$$

- S

$$= \frac{n}{4} \cot \frac{\vartheta}{2} \; a^2$$

$$= n \tan \frac{\vartheta}{2} \; r^2$$

$$= \frac{n}{2} \sin \vartheta \, R^2$$

$$= \frac{n}{2} a \, r$$

- $\vartheta = \frac{1}{n} 360^{o} = \frac{2}{n} \pi^{r}$

- $\alpha = \frac{n-2}{2n} 360^{o} = \frac{n-2}{n} \pi^{r}$

- $\beta = \frac{1}{n} 360^{o} = \frac{2}{n} \pi^{r}$

- $\gamma = \frac{n+2}{2n} 360^{o} = \frac{n+2}{n} \pi^{r}$

- $\delta = \frac{n-2}{4n} 360^{o} = \frac{n-2}{2n} \pi^{r}$

□ for any integer $n \geq 3$
consider a regular $n-$gon P_n
with circumradius R;
the n midpoints of the side - spanned arcs
of the circumcircle of P_n
together with the n vertices of P_n
form the 2n vertices of a regular 2n - gon P_{2n};
the side a_n of P_n
&
the side a_{2n} of P_{2n}
are related by the formula

$$a_{2n} = \sqrt{2R^2 - R\sqrt{4R^2 - a_n{}^2}}$$

□ for a given circle with diameter d

let

- $n \in$ var ranging over the integers ≥ 3
- I_n = the perimeter
  of the inscribed regular n - gon
- C_n = the perimeter
  of the circumscribed regular n - gon

then

- I_{2n} = the geometric mean of I_n and C_{2n}
  ie
  $$I_{2n} = \sqrt{I_n C_{2n}}$$
- C_{2n} = the harmonic mean of I_n and C_n
  ie
  $$\frac{1}{C_{2n}} = \frac{1}{2}\left(\frac{1}{I_n} + \frac{1}{C_n}\right)$$

• the classical polygonal method

of calculating π is based on the limits

$$\exists \lim_{n\to\infty} I_n = \text{circumference of circle} = \pi d$$

&

$$\exists \lim_{n\to\infty} C_n = \text{circumference of circle} = \pi d$$

&

$$I_3 < I_4 < I_5 < \cdots \rightarrow \pi d \leftarrow \cdots < C_5 < C_4 < C_3$$

☐ Gauss proved in 1796 at the age of nineteen that
for an integer n greater than or equal to 3
the regular polygon of n sides
is constructible by Platonic tools
iff
n is representable as
the product of
a nonnegative integer power of 2
and
distinct Fermat primes

☐ the Platonic tools
are
the unmarked straightedge
&
the collapsible compasses

☐ the nth Fermat number F_n
where n is a nonnegative integer
is defined to be
1 more than 2 to the 2 to the nth power
viz

$$F_n = 2^{2^n} + 1$$

□ the only known (2001) Fermat primes
are the first five Fermat numbers

$F_0 = 3$

$F_1 = 5$

$F_2 = 17$

$F_3 = 257$

$F_4 = 65537$

it is known that
$F_5 = 42949\ 67297$
is composite;
many other Fermat numbers
are known to be composite;
it is not known (2001) whether:

- all Fermat numbers F_n with index n at least 5 are composite

- there are infinitely many composite Fermat numbers

- there are infinitely many prime Fermat numbers

☐ the Platonically constructible regular n-gons
where n is less than or equal to 100
are given by
the following 24 values of n:
3, 4, 5, 6, 8,
10, 12, 15, 16, 17,
20, 24,
30, 32, 34,
40, 48,
54,
60, 64, 68,
80, 85,
96

☐ Gauss calculated that

$$\cos\frac{360^{\circ}}{17}$$

$$=$$

$$\frac{1}{16}\Big(-1+\sqrt{17}+\sqrt{34-2\sqrt{17}}$$
$$+2\sqrt{17+3\sqrt{17}-2\sqrt{34+2\sqrt{17}}-\sqrt{34-2\sqrt{17}}}\;\Big)$$

R. for any positive integer n

• the set of the nth roots of unity

= the set of the n complex nth roots of unity

= the set of the n roots of the nth degree

complex polynomial equation

$z^n = 1$

wh z $\in$ complex variable

= the set of the n complex numbers

$$\exp k\frac{2\pi}{n} = \cos k\frac{2\pi}{n} + i \sin k\frac{2\pi}{n}$$

wh k $\in$ int st $0 \le k \le n-1$

• if $n \ge 3$, then

the above set is the set of vertices

of a regular n - gon

with a vertex at 1,

with center at the origin,

with radius 1,

with central angle $\frac{2\pi}{n} = \frac{360^o}{n}$

GG65-37

☐ for the equilateral triangle P_3

- $n = 3$

- $a = 2\sqrt{3}\,r = \sqrt{3}\,R$

- $r = \dfrac{\sqrt{3}}{6}a = \dfrac{1}{2}R$

- $R = \dfrac{\sqrt{3}}{3}a = 2r$

- $p = 3a = 6\sqrt{3}\,r = 3\sqrt{3}\,R$

- $S = \dfrac{\sqrt{3}}{4}a^2 = 3\sqrt{3}\,r^2 = \dfrac{3\sqrt{3}}{4}R^2 = \dfrac{3}{2}ar$

- $\vartheta = 120^{o} = \frac{2\pi^{r}}{3}$
- $\alpha = 60^{o} = \frac{\pi^{r}}{3}$
- $\beta = 120^{o} = \frac{2\pi^{r}}{3}$
- $\gamma = 300^{o} = \frac{5\pi^{r}}{3}$
- $\delta = 30^{o} = \frac{\pi^{r}}{6}$

□ for the square P_4

- $n = 4$

- $a = 2r = \sqrt{2}R$

- $r = \frac{1}{2}a = \frac{\sqrt{2}}{2}R$

- $R = \frac{\sqrt{2}}{2}a = \sqrt{2}r$

- $p = 4a = 8r = 4\sqrt{2}R$

- $d = d_2 = \sqrt{2}a = 2\sqrt{2}r = 2R$

- $d:a = \sqrt{2}$

  = the simplest irrational real number

- $S = a^2 = 4r^2 = 2R^2 = 2ar$

GG65-40

- $\vartheta = 90^{o} = \frac{\pi^{r}}{2}$

- $\alpha = 90^{o} = \frac{\pi^{r}}{2}$

- $\beta = 90^{o} = \frac{\pi^{r}}{2}$

- $\gamma = 270^{o} = \frac{3\pi^{r}}{2}$

- $\delta = 45^{o} = \frac{\pi^{r}}{4}$

□ for the regular pentagon P_5

- $n = 5$

- $a = 2\sqrt{5 - 2\sqrt{5}}\, r = \frac{1}{2}\sqrt{10 - 2\sqrt{5}}\, R$

- $r = \frac{1}{10}\sqrt{25 + 10\sqrt{5}}\, a = \frac{1}{4}\left(1 + \sqrt{5}\right) R$

- $R = \frac{1}{10}\sqrt{50 + 10\sqrt{5}}\, a = \left(\sqrt{5} - 1\right) r$

- $p = 5a = 10\sqrt{5 - 2\sqrt{5}}\, r = \frac{5}{2}\sqrt{10 - 2\sqrt{5}}\, R$

- $d = d_2 = d_3$

$= \frac{1}{2}\left(1 + \sqrt{5}\right) a$

$= \sqrt{10 - 2\sqrt{5}}\, r$

$= \frac{1}{2}\sqrt{10 + 2\sqrt{5}}\, R$

- $d:a = \frac{1}{2}\left(1+\sqrt{5}\right)$

$= 1.47433\ 57156+ \ = \varphi = $ the golden ratio

- any two interiorly intersecting diagonals

cut segments on each other
that are in golden ratio

- S

$$= \frac{1}{4}\sqrt{25+10\sqrt{5}}\ a^2$$

$$= 5\sqrt{5-2\sqrt{5}}\ r^2$$

$$= \frac{5}{8}\sqrt{10+2\sqrt{5}}\ R^2$$

$$= \frac{5}{2}ar$$

- $\vartheta = 72^o = \frac{2\pi^r}{5}$

- $\alpha = 108^o = \frac{3\pi^r}{5}$

- $\beta = 72^o = \frac{3\pi^r}{5}$

- $\gamma = 252^o = \frac{7\pi^r}{5}$

- $\delta = 54^o = \frac{3\pi^r}{10}$

☐ for the regular hexagon P_6

- $n = 6$

- $a = \frac{2\sqrt{3}}{3} r = R$

- $r = \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{2} R$

- $R = a = \frac{2\sqrt{3}}{3} r$

- $p = 6a = 4\sqrt{3}\, r = 6R$

- $d_2 = d_4 = \sqrt{3}\, a = 2r = \sqrt{3}\, R$

- $d_3 = 2a = \frac{4\sqrt{3}}{3} r = 2R$

- $S = \frac{3\sqrt{3}}{2} a^2 = 2\sqrt{3}\, r^2 = \frac{3\sqrt{3}}{2} R^2 = 3ar$

- $\vartheta = 60^{o} = \frac{\pi^{r}}{3}$

- $\alpha = 120^{o} = \frac{2\pi^{r}}{3}$

- $\beta = 60^{o} = \frac{\pi^{r}}{3}$

- $\gamma = 240^{o} = \frac{4\pi^{r}}{3}$

- $\delta = 60^{o} = \frac{\pi^{r}}{3}$

□ for the regular octagon P_8

- $n = 8$

- $a = 2\left(\sqrt{2}-1\right) r = \sqrt{2-\sqrt{2}}\, R$

- $r = \frac{1}{2}\left(1+\sqrt{2}\right) a = \frac{1}{2}\sqrt{2+\sqrt{2}}\, R$

- $R = \frac{1}{2}\sqrt{4+2\sqrt{2}}\, a = \sqrt{4-2\sqrt{2}}\, r$

- $p = 8a = 16\left(\sqrt{2}-1\right) r = 8\sqrt{2-\sqrt{2}}\, R$

- $d_2 = d_6$

$$= \sqrt{2+\sqrt{2}}\,a = 2\sqrt{2-\sqrt{2}}\,r = \sqrt{2}\,R$$

- $d_3 = d_5$

$$= \left(1+\sqrt{2}\right)a = 2\,r = \sqrt{2+\sqrt{2}}\,R$$

- d_4

$$= \frac{1}{2}\sqrt{2+\sqrt{2}}\,a = 2\sqrt{4-2\sqrt{2}}\,r = 2\,R$$

- S

$$= 2\left(1+\sqrt{2}\right)a^2 = 8\left(\sqrt{2}-1\right)r^2 = 2\sqrt{2}\,R^2 = 4\,a\,r$$

- $\vartheta = 45^o = \dfrac{\pi^r}{4}$

- $\alpha = 135^o = \dfrac{3\pi^r}{4}$

- $\beta = 45^o = \dfrac{\pi^r}{4}$

- $\gamma = 225^o = \dfrac{5\pi^r}{4}$

- $\delta = 67^o\ 30' = \dfrac{3\pi^r}{8}$

☐ for the regular decagon P_{10}

- $n = 10$

- $a = \frac{2}{5}\sqrt{25-10\sqrt{5}}\, r = \frac{1}{2}\left(\sqrt{5}-1\right)R$

- $r = \frac{1}{2}\sqrt{5+2\sqrt{5}}\, a = \frac{1}{4}\sqrt{10+2\sqrt{5}}\, R$

- $R = \frac{1}{2}\left(1+\sqrt{5}\right)a = \frac{1}{5}\sqrt{50-10\sqrt{5}}\, r$

- $p = 10a = 4\sqrt{25-10\sqrt{5}}\, r = 5\left(\sqrt{5}-1\right)R$

- $R:a = \frac{1}{2}\left(1+\sqrt{5}\right) = \varphi =$ the golden ratio

- $d_2 = d_8$

$$= \frac{1}{2}\sqrt{10+2\sqrt{5}}\ a = \frac{1}{2}\left(\sqrt{5}-1\right) r = \frac{1}{2}\sqrt{10-2\sqrt{5}}\ R$$

- $d_3 = d_7$

$$= \frac{1}{2}\left(3+\sqrt{5}\right) a = \frac{1}{5}\sqrt{50+10\sqrt{5}}\ r = \frac{1}{2}\left(1+\sqrt{5}\right) R$$

- $d_4 = d_6$

$$= \sqrt{5+2\sqrt{5}}\ a = 2r = \frac{1}{2}\sqrt{10+2\sqrt{5}}\ R$$

- d_5

$$= \left(1+\sqrt{5}\right) a = \frac{2}{5}\sqrt{50-10\sqrt{5}}\ r = 2R$$

- $d_3 : R = \frac{1}{2}\left(1+\sqrt{5}\right) = \varphi =$ the golden ratio

- S

$$= \frac{5}{2}\sqrt{5+2\sqrt{5}}\,a^2$$

$$= 2\sqrt{25-10\sqrt{5}}\,r^2$$

$$= \frac{5}{4}\sqrt{10-2\sqrt{5}}\,R^2$$

$$= 5ar$$

- $\vartheta = 36^{o} = \frac{\pi^{r}}{5}$

- $\alpha = 144^{o} = \frac{4\pi^{r}}{5}$

- $\beta = 36^{o} = \frac{\pi^{r}}{5}$

- $\gamma = 216^{o} = \frac{6\pi^{r}}{5}$

- $\delta = 72^{o} = \frac{2\pi^{r}}{5}$

D. star polygons

let

- $n \in$ int ≥ 5
- $(V, P) \in$ n - gon

wh $V = \langle A_0, A_1, \cdots, A_{n-1} \rangle$

- $k \in$ int

st $2 \leq k \leq n - 2$ & k is coprime to n

then

the star polygon

of order n & of span k

in (V, P)

$=_{dn}$ $SP(n, k; (V.P))$

$=_{df}$ the n - gon whose vertex cycle is

$\langle A_0, A_k, A_{2k}, A_{3k}, \cdots, A_{(n-1)k} \rangle$

wh the subscripts are to be reduced mod n

D. a regular star polygon

$=_{df}$ a star polygon in a regular polygon

☐ names of some n-gons starting with n = 3

n	name of n-gon	
3	triangle	
4	quadrilateral	= quadrangle
5	pentagon	
6	hexagon	
7	heptagon	
8	octagon	
9	nonagon	= enneagon
10	decagon	
11	undecagon	= hendecagon
12	dodecagon	= duodecagon
13	tridecagon	
14	tetradecagon	
15	pentadecagon	
16	hexadecagon	
17	heptadecagon	
18	octadecagon	
19	enneadecagon	
20	icosagon	
30	triacontagon	
40	tetracontagon	
50	pentacontagon	
60	hexacontagon	
70	heptacontagon	
80	octacontagon	
90	enneacontagon	
100	hectogon	
1000	chiliagon	
10 000	myriagon	

□ to make up a name of the n-gon
where n is an integer such that $20 < n < 100$
but n is not a multiple of 10, take
prefix + suffix = n

prefix
20 icosikai
30 triacontakai
40 tetracontakai
50 pentacontakai
60 hexacontakai
70 heptacontakai
80 octacontakai
90 enneacontakai

plus

suffix
1 henagon
2 digon
3 trigon
4 tetragon
5 pentagon
6 hexagon
7 heptagon
8 octagon
9 enneagon

note: the polygon words on this & the preceding page
are rooted in Greek (and some Latin) number words
& γωνια (Greek) = angle & και (Greek) = and GG65-56

☐ some names of regular star polygons

• pentagram
< 5, 2 >

• septagrams
< 7, 2 > & < 7, 3 >

• octogram
< 8, 3 >

• nonagrams = enneagrams
< 9, 2 > & < 9, 4 >

• decagram
< 10, 3 >

• note: the hexagram is not a star polygon
but rather the union of
two opposed congruent equilateral triangles
with common center and parallel sides;
the hexagram is formed also
by extending the sides of a regular hexagon;
hexagram
= Star of David
= Shield of David
= Magen David
= Mogen David
= Solomon's Seal

□ more notions about polygons

• a polygon is said to be
plane or skew
according as
its vertices
do or do not
lie in a single plane

• a polygon is said to be
oriented or nonoriented
according as
its vertex cycle is
an oriented or a nonoriented
cyclic sequence

• the preceding pages
are confined to
plane polygons
&
may be considered to be confined to
nonoriented polygons

• a polygon is said to be
self - intersecting or non - self - intersecting
according as
the intersection of
some or no
pair of sides
contains an interior point of a side
eg
regular polygons
are
non - self - intersecting
&
star polygons
are
self - intersecting

• a sagitta of a regular polygon
$=_{df}$ the line segment
from the midpoint of a side of the polygon
to the midpoint of the arc of the circumcircle
bounded by the endpoints of the side;
sagitta $=_{pr}$ sa - JIT - uh
is the Latin word for ' arrow' ;
for an arc with chord
think of an arrow
from the midpoint of the straight taut string = chord
to the midpoint of the bow = arc
at rest in loading position;
a regular n - gon ($n \in$ int ≥ 3)
has exactly n sagittas,
one to a side

• note: circumradius = apothem + sagitta

☐ cyclic sequences

• an / a oriented / nonoriented cyclic sequence
is defined to be a certain kind of a set of sequences
eg
the oriented cyclic sequence
< a, b, c, d >
$=_{df}$ the set of four sequences
{(a, b, c, d),
(b, c, d, a),
(c, d, a, b),
(d, a, b, c)}
&
the nonoriented cyclic sequence
<< a, b, c, d >>
$=_{df}$ < a, b, c, d > ∪ < d, c, b, a >