A Few Good
Distance-Rate-Time Problems

#62 of Gottschalk's Gestalts

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GG62-2

□ consider a moving particle,
mathematically represented by a point,
in uniform (= constant) speed
along a straight line (= in rectilinear motion)
or
along a smooth curve (= in curvilinear motion)
for a specified finite amount of time

• distance
= distance the particle has moved
along its orbit / path
$=_{dn}$ d ← d̲istance

• rate
= the constant rate of speed of the particle
along its orbit / path
$=_{dn}$ r ← r̲ate

• time
= the time of duration of the particle's motion
along its orbit / path
$=_{dn}$ t ← t̲ime

☐ it is clear from physical considerations that:

• distance equals rate times time

$$d = r \times t$$

$$d = rt$$

• rate equals distance divided by time

$$r = d \div t$$

$$r = \frac{d}{t}$$

• time equals distance divided by rate

$$t = d \div r$$

$$t = \frac{d}{r}$$

☐ given any two of the quantities

distance = d

rate = r

time = t

the third is uniquely determined by one of the three above formulas

☐ below is a list of five distance - rate - time problems that are nonroutine challenges of varing difficulty

P1. Rowing Upsteam & Downstream
A person can row a boat six miles per hour
in still water.
In a river where the current is two miles per hour,
it takes him thirty minutes longer
to row a certain distance upstream
than
to row the same distance downstream.
Find
how long it takes him to row upstream,
how long to row downstream,
and how many miles he rows.

S1. rowing upstream & downstream
• let d denote the distance rowed in one direction

• he rows 6 - 2 = 4 mph upstream
&
he rows 6 + 2 = 8 mph downstream

• hence
the time rowing upstream is $\frac{d}{4}$ hours
&
the time rowing downstream is $\frac{d}{8}$ hours

• then

$$\frac{d}{4} = \frac{d}{8} + \frac{1}{2}$$

&

d = 4 miles

• time to row upstream is one hour;
time to row downstream is thirty minues;
total distance rowed is eight miles

P2. The Rowboat & the Beach Ball
A person in a rowboat on a calmly flowing river
drops a beach ball in the river
and then rows upstream for ten minutes.
He immediately turns around and rows downstream
to overtake and pick up the ball.
He then notices that
the ball has floated downstream
for exactly one mile from the drop-off point.
How fast is the river flowing?

S2. the rowboat & the beach ball
• thinking of relative motion on the river itself,
the oarsman drops the ball on a stationary stream
and rows away for 10 minutes ;
he then rows back to the ball
and this last trip must also take 10 minutes

• in that 20 minute period
the river has moved one mile;
hence
the current of the river
is 3 miles per hour

P3. Two Trains & a Fly
Two trains are heading toward one another on parallel tracks
and are originally
a distance apart of 200 miles.
One train is traveling 40 miles per hour.
The other train is traveling 60 miles per hour.
A fast indefatigible fly starting from one train
at the beginning flies back and forth
at a speed of 80 miles per hour
from one train to the other train continuously,
first touching one train and then the other without rest
until the trains meet.
How far did the fly fly?

S3. two trains & a fly
• the trains meet in 2 hours;
the fly then flies for 2 hours

• the fly files at a speed of 80 mph

• hence the fly flies 2 times 80 = 160 miles

P4. The Passenger Train & the Freight Train
A passenger train is x times faster than a freight train.
The passenger train takes x times as long
to overtake the freight train
when going in the same direction
as it takes the two trains to pass
when going in opposite directions.
Find x.

S4. the passenger train & the freight train
• assume the speed of the freight train to be unity
& assume the sum of the lengths of the trains to be unity;
this then determines the unit of time;
this simpifies the algebraic calculation

• their relative speed when going in the same direction is $x-1$;
their relative speed when going in opposite directions is $x+1$;
the overtaking/passing distance
in the same/opposite direction(s) is 1;

• the passenger train takes $\frac{1}{x-1}$ time units to overtake in the same direction;
the pasenger train takes $\frac{1}{x+1}$ time units to pass in the opposite direction

• hence

$$\frac{1}{x-1} = \frac{x}{x+1}$$

&

$$x = 1+\sqrt{2} = 2.414213562+$$

P5. The Army & the Courier
An army ten miles long is marching along
a winding mountain road.
A motorcycle courier delivers a message
from the rear of the army to the front of the army
and immediatley returns to the rear.
He notices that when he returns to the rear
that he is at the point where the front of the army was
when he started out.
How far did the courier travel?

S5. the army & the courier
• for two uniform motions
during two time intervals t and t',

$$d_1 = r_1 t$$

$$d_2 = r_2 t$$

$$d_1' = r_1 t'$$

$$d_2' = r_2 t'$$

whence

$$\frac{d_1}{d_2} = \frac{r_1}{r_2}$$

$$\frac{d_1'}{d_2'} = \frac{r_1}{r_2}$$

&

$$\frac{d_1}{d_2} = \frac{d_1'}{d_2'}$$

• let the length of the army be the unit of length to simplify the little algebraic manipulations

• let x be the distance
the courier traveled forward
from the position of the army front at outset
to the position of the army front at delivery;
the total distance covered by the courier
is $2x + 1$

• using the two time intervals during which
the courier traveled
from original back to delivery front
&
from delivery front to final back (= original front position)
and also using
the proportion established above

$$\frac{1+x}{x} = \frac{x}{1-x}$$

whence

$$x = \frac{\sqrt{2}}{2}$$

&

$$2x+1 = 1+\sqrt{2} = 2.414213562+$$

• hence
the total distance the courier traveled is
$10\left(1+\sqrt{2}\right) = 24.14213562+$ miles