Initial Letters Provide Literal Notation

#49 of Gottschalk's Gestalts

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☐ Names of Notions Nominate Notation

• the initial letters of names of objects
often provide
good efficient suggestive easy-to-remember
literal notation
for the notions under consideration;
that general notational principle
is illustrated in several ways in the following

☐ first letters are good symbols

it has long been generally recognized that
taking the first letter (lowercase or capital) of the name
of a mathematical object (individual or species),
or first letter of the principal word in a name phrase,
as a symbol/sign (constant or variable) for this object
is often sound notational practice
eg

- a = angle
- A = algebra
- A_n = the alternating group on n objects
- A = angle
- A = area

- b = base length
- B_n or B^n = n-ball
- B = Banach space
- B = base area
- $B(n)$ = the nth Bell number
- B_n = the nth Bernoulli number
- B_r = the bounding r-cycle group
- B^r = the cobounding r-cocycle group

- c = cardinal of the continuum
- c = constant
- C_r = the r-chain group
- C^r = the r-cochain group
- C = circumference
- ${}_nC_r$ = the number of combinations of n things taken r at a time
- C = complex
- C = constant
- C = curve
- d = diameter
- d = difference
- d = differential/derivative
- D = derivative
- D = discriminant
- D = domain

- e = eccentricity
- e = base of exponential function
- E = entropy
- f_n = the nth Fibonacci number
- f = function
- F = truth-value falsity
- F_n = the nth Fermat number
- F = field
- F = function

- g = genus
- G = group
- h = height
- h = homeomorphism
- H = Hamiltonian (from Hamilton)
- H_p or H^p = the Hardy space of index p
- H = Hessian (from Hesse)
- H_n = the nth harmonic number
- H = Hilbert space
- H_r = the r-homology group
- H^r = the r-cohomology group

- i = imaginary unit
- i = index
- I = identity matrix
- I = indicator
- I = integral
- I = interval
- J = Jacobian (from Jacobi)
- k = constant (phonetic value)
- K = complex (phonetic value)
- K = knot

- l_p or l^p = the Lebesgue sequence space of index p (note the script lowercase el)
- L = Lagrangian (from Lagrange)
- L_p or L^p = the Lebesgue function space of index p
- L = length
- m = mean
- m = measure
- m = modulus
- m = moment
- M = Turing machine
- M = manifold
- M = matrix
- M_n = the nth Mersenne number
- M = module
- M = monoid

- n = number
- N = norm
- N = number
- O = origin
- p = prime number
- p = proposition
- ${}_nP_r$ = the number of permutations of n things taken r at a time
- P = point
- P = polynomial
- q = quaternion
- Q = quadrant

- r = radial distance
- r = radian
- r = radius
- r = ratio
- R = range
- R = region
- R = relation
- s = semiperimeter
- s = side
- s = subtending arc
- S = space
- S^n = n-sphere
- S = surface
- S = surface area

- t = time
- T = tensor
- T = transformation
- T = truth-value truth
- **u** = unit vector
  (note the boldface lowercase yu)
- v = velocity
- V = Vandermonde determinant
- V = variation
- V = vector space
- V = volume
- w = weight
- w = width
- W = Wronskian (from Wronski)

□ basic notation for sets / systems of numbers;
this notation uses
capital English letters in the open - face style;
this notation is now more - or - less
universally adopted in denoting
the main line of number systems
viz
$\mathbb{P} \subset \mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C} \subset \mathbb{H} \subset \mathbb{O}$

• $\mathbb{P}$ = the set of all positive integers
= the set of positive integers
= the positive integer set
= the positive integers
= the semiring of positive integers

• $\mathbb{N}$ = the set of all nonnegative integers
= the set of nonnegative integers
= the nonnegative integer set
= the nonnegative integers
= the semiring of nonnegative integers

• $\mathbb{Z}$ = the set of all integers

= the set of integers

= the integer set

= the integers

= the ring of integers

• $\mathbb{Q}$ = the set of all rational numbers

= the set of rational numbers

= the rational number set

= the rational numbers

= the set of all rationals

= the set of rationals

= the rationals

= the field of rational numbers

- $\mathbb{R}$ = the set of all real numbers

= the set of real numbers

= the real number set

= the real numbers

= the set of all reals

= the set of reals

= the reals

= the field of real numbers

= the field of reals

- $\mathbb{C}$ = the set of all complex numbers

= the set of complex numbers

= the complex number set

= the complex numbers

= the field of complex numbers

- $\mathbb{H}$ = the set of all quaternions

= the set of quaternions

= the quaternion set

= the quaternions

= the division ring of quaternions

- $\mathbb{O}$ = the set of all octonions

= the set of octonions

= the octonion set

= the octonions

= the nonassociative noncommutative real linear algebra of octonions

- origin of notation

$\mathbb{P} \leftarrow$ positive

$\mathbb{N} \leftarrow$ nonnegative, natural, number

$\mathbb{Z} \leftarrow$ die Zahl (German) = number

$\mathbb{Q} \leftarrow$ quotient

$\mathbb{R} \leftarrow$ real

$\mathbb{C} \leftarrow$ complex

$\mathbb{H} \leftarrow$ Hamilton = discoverer / inventor of quaternions

$\mathbb{O} \leftarrow$ octonion

□ the field of all complex algebraic numbers
= the field of complex algebraic numbers
= the complex algebraic number field
$=_{dn}$ $\mathbb{A}$
$=_{rd}$ (open cap) ay
wh
$\mathbb{A}$ ← the initial letter of ' algebraic'

□ the field of all real algebraic numbers
= the field of real algebraic numbers
= the real algebraic number field
= $\mathbb{A} \cap \mathbb{R}$
$=_{dn}$ $\mathbb{A}_r$
wh
$\mathbb{A}_r \leftarrow \mathbb{A}$ and the initial letter of ' real'

□ the ring of all Gaussian integers

= the ring of Gaussian integers

= the Gaussian integer ring

$=_{df} \{m + i\,n \mid m, n \in \mathbb{Z}\}$

$= \mathbb{Z} + i\,\mathbb{Z}$

$=_{dn} \mathbb{G}$

$=_{rd}$ (open cap) gee

wh

$\mathbb{G} \leftarrow$ the initial letter of 'Gauss'

□ the field with exactly p^n elements
wh $p \in$ prime & $n \in$ pos int

= the field with p^n elements

= the field of order p^n

$=_{dn}$ $\mathbb{F}(p^n)$

$=_{rd}$ (open cap) ef of p^n

wh

$\mathbb{F} \leftarrow$ the common initial letter of 'finite field'

note:

a field with only finitely many elements

= a field with exactly a prime power p^n of elements

= a finite field

= a Galois field

□ euclidean space of dimension n

wh n ∈ nonneg int

= n - dimensional euclidean space

= euclidean n - space

$=_{dn}$ $\mathbb{E}^n$

$=_{rd}$ (open cap) ee (super) n

wh

$\mathbb{E}$ ← the initial letter of ' euclidean' (from Euclid)

□ projective space of dimension n

wh n $\in$ nonneg int

= n - dimensional projective space

= projective n - space

$=_{dn}$ $\mathbb{P}^n$

$=_{rd}$ (open cap) pe (super) n

wh

$\mathbb{P}$ $\leftarrow$ the initial letter of 'projective'

□ the unit circle

= the circle group

$=_{dn}$ $\mathbb{T}$

$=_{rd}$ (open cap) tee

wh

$\mathbb{T}$ ← the initial letter of ' torus'

the circle being the 1 - torus

□ the n - dimensional torus

wh n ∈ nonneg int

= the n - torus

= the n - toral group

$=_{dn}$ $\mathbb{T}^n$

$=_{rd}$ (open cap) tee (super) n

wh

$\mathbb{T}$ ← the initial letter of ' torus'

☐ sometimes
initial letters of words
from other languages
make notational contributions;
here are four examples from German:

• e = unit
from the German word
die Einheit = unit/unity

• U = neighborhood
from the German word
die Umgebung = neighborhood

• Z_r = the r-cycle group
& Z^r = the r-cocycle group
from the German word
der Zyklus = cycle

• $\mathbb{Z}$ = the set of integers
from the German word
die Zahl = number

☐ sometimes
the corresponding letter in Greek
is used for the notation
instead of
the first letter of the English name;
the original word
may have itself been in Latin or Greek
eg

- α = angle

- ε = initial letter of the Greek word εστι
(see the Latin est = is inside?)
meaning 'is' and used to stand for
'is an element of'

- κ = curvature

- λ = Lagrange multiplier

- μ = mean

- π = periphery = circumference
(of a circle with unit diameter)

- $\pi(x)$ = prime-counting function

- ρ = radius of curvature

- σ = simplex

- τ = torsion

- φ = function

- φ = denotation of the golden ratio;

so chosen in honor of
Phidias (Φειδιας) of Athens
fl ca 490-430 BCE
Greek
greatest sculptor of ancient Greece;
supervised construction of Parthenon

- χ = characteristic
- Δ = difference
- Δ = discriminant
- Π = product/production
- Σ = sum/summation
- Φ = function

☐ apparently
for some functions
the letter denotation came first
and the name came from the letter
eg

- the Kronecker delta = δ_{ij} eg
- the Dirac delta function = $\delta(x)$
- the Riemann zeta function = $\zeta(z)$
- the Möbius mu function = $\mu(n)$
- the Euler phi function = $\varphi(n)$
- the Euler beta function = $B(x, y)$
- the Euler gamma function = $\Gamma(x)$

□ notation for Borel sets & their classes

Δ letters for the classes of closed sets & open sets
in a topological space

• F = the class of closed sets
which comes from
the initial letter of the French word
fermé = closed

• G = the class of open sets
which comes from
the initial letter of the German word
das Gebiet = region

Δ letters for set-theoretic operators on classes of sets

• σ = the countable union operator for a class of sets
which is the lowercase form of
the Greek letter sigma Σ σ
which is suggested by the initial letter of
the following words
sum (English)
= la somme (French)
= die Summe (German)
= summa (Latin)
since ess & sigma correspond
in sound & transliteration

• δ = the countable intersection operator for a class of sets
which is the lowercase form of
the Greek letter delta Δ δ
which is suggested by the initial letter of
the German word
der Durchschnitt = intersection
since dee & delta correspond
in sound & transliteration

Δ the classes of Borel sets of index $< \omega$

$$F \xleftrightarrow{\text{dual}} G$$

F_σ G_δ

$F_{\sigma\delta}$ $G_{\delta\sigma}$

$F_{\sigma\delta\sigma}$ $G_{\delta\sigma\delta}$

etc

wh

these symbols are read as follows:

F = ef

F_σ = ef sigma

$F_{\sigma\delta}$ = ef sigma delta

etc

G = gee

G_δ = gee delta

$G_{\delta\sigma}$ = gee delta sigma

etc

☐ to write a bold-face lowercase or capital letter, underline
eg

- boldface a = **a** = $\underline{a}$
- boldface A = **A** = $\underline{A}$

note: letters denoting vectors & matrices are often printed in boldface type

☐ capital script letters are sometimes useful;
here are a few examples

- script B = $\mathcal{B}$ = filter-base
- script C = $\mathcal{C}$ = Cauchy filter/filter-base
- script C = $\mathcal{C}$ = cluster = class of sets
- script F = $\mathcal{F}$ = filter
- script I = $\mathcal{I}$ = (coefficient of) imaginary part of
- script N = $\mathcal{N}$ = neighborhood filter
- script P = $\mathcal{P}$ = power set of
- script R = $\mathcal{R}$ = real part of
- script T = $\mathcal{T}$ = topology

note: cap script 'letter'
may be used to denote
the system whose base is denoted by
cap Roman 'letter'

□ typographically ambiguous letters

Δ the three English letter forms of
lowercase el = l
capital oh = O
lowercase oh = o
are typo-ambiguous in that
the first letter resembles
the numeral one = 1
&
the second two letters resemble
the numeral zero = 0
& a pictograph for a circle
which is a circle;
thus their use requires caution

• if lowercase el is to be suggestively used
for 'length' say,
it is to be recommended that
the script lowercase el
be used to distinguish it from
the numeral one = 1

• the use of cap oh O
for the origin of a coordinate system
is congenial
because
oh is the initial letter of 'origin'
and all the coordinates of the origin are
zero

• the use of cap oh O
for a general operation
with prefix notation
is congenial;
observe
O(x)
O(x, y)
O(x, y, z)
etc
wh O is from the initial letter of 'operation'

• the use of lowercase oh o (suspended)
for a general binary operation
with infix notation
is congenial;
observe
x o y
wh o is the initial letter of 'operation'

Δ there is typo-ambiguity
between thirteen capital letters
of the Greek alphabet
and capital letters
of the English /Latin alphabet
viz

- A = cap Greek alpha = cap English ay
- B = cap Greek beta = cap English bee
- E = cap Greek epsilon = cap English ee
- Z = cap Greek zeta = cap English zee
- H = cap Greek eta = cap English aitch
- I = cap Greek iota = cap English eye
- K = cap Greek kappa = cap English kay
- M = cap Greek mu = cap English em
- N = cap Greek nu = cap English en
- O = cap Greek omicron = cap English oh
- P = cap Greek rho = cap English pe
- T = cap Greek tau = cap English tee
- X = cap Greek chi = cap English ex

Δ there is also typo-ambiguity
with four lowercase letters:

• lowercase Greek kappa κ
is similar to
but not identical with
lowercase English kay k

• ο = lowercase Greek omicron
= lowercase English oh

• terminal lowercase Greek sigma ς
is similar to
but not identical with
lowercase English ess s

• lowercase Greek chi χ
is similar to
but not identical with
lowercase English ex x

☐ some one-word section headings
may be conveniently abbreviated by
the capitalized initial letter
followed by a period
eg

- Axiom. = A.
- Comment. = C.
- Corollary. = K. (phonetic value)
- Definition. = D.
- Example. = E.
- Lemma. = L.
- Note. = N.
- Proof. = P.
- Question. = Q.
- Remark. = R.
- Theorem. = T.

□ to say that a letter notation
comes from the initial letter of a certain word
may be fully & historically correct only if
the etymology of the word is considered
as part of the word;
here are two examples

• Euler was the first to use i as the imaginary unit
whose square is -1;
that occurred in 1777;
since he wrote in Latin,
to Euler i would be
the initial letter of the Latin word
imaginarius = imaginary
from which the English word 'imaginary'
descends

• the notation π for the circle ratio
was first used by
William Jones
1675-1749
Welsh
applied mathematician,
mathematics teacher & expositor;
that occurred in 1706;
π is the initial letter of the Greek word
περιφερω = to carry around
from which the English word 'periphery'
descends