The Four Functions in the Del

#46 of Gottschalk's Gestalts

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□ the del operator

• the del operator

$=_{dn} \nabla$

$=_{rd}$ del $\leftarrow$ inverted delta

$=_{df}$ the partial derivative operator

$$\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) = \mathbf{i} \frac{\partial}{\partial x} + \mathbf{j} \frac{\partial}{\partial y} + \mathbf{k} \frac{\partial}{\partial z}$$

which converts a scalar field
over a region in 3 - space
into a 3 - vector field
over that region
wh x, y, z are
3 - dimensional rectangular coordinate variables

• the del

= the inverted delta

= ∇

is also called the nabla

presumably because of its resemblance in shape to

the ancient Hebrew harp,

a stringed instrument of ten or twelve strings

which has the name nabla & also the symbol ∇

□ the

• gradient	del	∇
• divergence	del dot	$\nabla \bullet$
• curl	del cross	$\nabla \times$
• laplacian	del square	∇^2

form

the four functions in the del
aka
the four del - based operators

- [ ] the table

• gradient	grad
• divergence	div
• curl	curl
• laplacian	lap

displays

the syllabus of single syllables
for the four del - based operators

☐ the four transformations of scalar / vector fields under the action of the del - based operators

the operator	operates upon a	to produce a
• gradient	scalar field	vector field
• divergence	vector field	scalar field
• curl	vector field	vector field
• laplacian	scalar field	scalar field

- [ ] geometrical / physical catchphrases for the information provided by the four del - based operators

- gradient points uphill

- divergence is emergence

- curl is swirl

- laplacian measures
local average value
minus
central value

☐ all four del - based operators
have highly significant uses in mathematics;
it hardly makes sense to ask
which is 'the most important' in mathematics;
they are all important;
however it has been claimed that the laplacian
is by far the most important differential operator
in mathematical physics;
here is a single example to help bolster that claim

□ the wave equation
is a second - order partial differential equation
using the laplacian & is here given in two notations

- $\nabla^2\varphi = \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}$

- $\Box\varphi = 0$

wh
$\varphi = \varphi(x, y, z, t)$
is a scalar - valued function
of position (x, y, z) & time t
and c is a constant;
the d' Alembertian □
is defined to be the partial differential operator

$$\Box = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$$