Initial Analogies:
Line in Plane, Plane in Space, Line in Space

#33 of Gottschalk's Gestalts

A Series Illustrating Innovative Forms
of the Organization & Exposition
of Mathematics
by Walter Gottschalk

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GG33-1 (32)

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GG33-2

□ given a straight line
in the plane
which is provided with
a rectangular xy - coordinate system

Δ standard notation
for the line:
real variables & real constants,
real 2 - vector variable & real 2 - vector constants

- x, y, t = real variables

- x = x - coordinate = abscissa

- y = y - coordinate = ordinate

- t = parameter

(wh t is suggestive of ' time')

• running point on line

= $P(x, y)$

• particular points on line

$P_0(x_0, y_0)$,

$P_1(x_1, y_1)$,

$P_2(x_2, y_2)$

• coordinate vector variable

= $\mathbf{r} = (x, y)$

• particular points on line

$\mathbf{r}_0 = (x_0, y_0)$,

$\mathbf{r}_1 = (x_1, y_1)$,

$\mathbf{r}_2 = (x_2, y_2)$

- a = x - intercept
- b = y - intercept
- m = slope
- p = normal distance from origin
- A, B, C = coefficients
- α = inclination
- ω = normal angle

- α' , β' = line direction angles
- $\cos\alpha'$, $\cos\beta'$ = line direction cosines
- l' , m' = line direction numbers
- α^*, β^* = normal direction angles
- $\cos\alpha^*$, $\cos\beta^*$ = normal direction cosines
- l^*, m^* = normal direction numbers

• **l** = line direction vector

= any nonzero vector parallel to the line

eg

$(\cos\alpha', \ \cos\beta')$ or $(l', \ m')$

• **n** = normal direction vector

= any nonzero vector normal to the line

eg

$(\cos\alpha^*, \ \cos\beta^*)$ or $(l^*, \ m^*)$

• **A** = (A, B) = coordinate coefficient vector

Δ standard scalar forms
of the canonical rectangular equations of the line

• general form

$$Ax + By + C = 0$$

• slope - intercept form

$$y = mx + b$$

• intercept form

$$\frac{x}{a} + \frac{y}{b} = 1$$

- first point - slope form

$$\frac{y - y_0}{x - x_0} = m$$

- second point - slope form

$$y - y_0 = m(x - x_0)$$

- first two - point form

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

- second two - point form

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

- two - point determinant form

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

• first normal form

$$x \cos \omega + y \sin \omega - p = 0$$

• second normal form

$$x \cos \alpha^* + y \cos \beta^* \pm p = 0$$

• point - normal - direction - cosines form

$$(x - x_0) \cos \alpha^* + (y - y_0) \cos \beta^* = 0$$

• point - normal - direction - numbers form

$$(x - x_0) l^* + (y - y_0) m^* = 0$$

• point - line - direction - cosines form

$$\frac{x - x_0}{\cos\alpha'} = \frac{y - y_0}{\cos\beta'}$$

• point - line - direction - numbers form

$$\frac{x - x_0}{l'} = \frac{y - y_0}{m'}$$

• parametric form

$$\begin{cases} x = x_0 + l'\, t \\ y = y_0 + m'\, t \end{cases}$$

Δ standard vector forms
of the canonical rectangular equations of the line

• general form

$$\mathbf{A} \cdot \mathbf{r} + C = 0$$

• two - point form

$$\left| \mathbf{r} - \mathbf{r}_1, \; \mathbf{r}_2 - \mathbf{r}_1 \right| = 0$$

• point - normal - direction form

$$(\mathbf{r} - \mathbf{r_0}) \cdot \mathbf{n} = 0$$

• point - line - direction form

$$\left| \mathbf{r} - \mathbf{r_0}, \; \mathbf{l} \right| = 0$$

• parametric form

$$\mathbf{r} = \mathbf{r_0} + \mathbf{l}\, t$$

□ given a plane
in 3 - space
which is provided with
a rectangular xyz - coordinate system

Δ standard notation
for the plane:
real variables & real constants,
real 3 - vector variable & real 3 - vector constants

- x, y, z, t, s = real variables

- x = x - coordinate = abscissa

- y = y - coordinate = ordinate

- z = z - coordinate = altitude

- t, s = parameters

(wh t is suggestive of ' time'
& the alphabetic predecessor s of ' space')

• running point on plane

$= P(x, y, z)$

• particular points on plane

$P_0(x_0, y_0, z_0)$,

$P_1(x_1, y_1, z_1)$,

$P_2(x_2, y_2, z_2)$,

$P_3(x_3, y_3, z_3)$

• coordinate vector variable

$= \mathbf{r} = (x, y, z)$

• particular points on plane

$\mathbf{r}_0 = (x_0, y_0, z_0)$,

$\mathbf{r}_1 = (x_1, y_1, z_1)$,

$\mathbf{r}_2 = (x_2, y_2, z_2)$,

$\mathbf{r}_3 = (x_3, y_3, z_3)$

- a = x - intercept
- b = y - intercept
- c = z - intercept
- p = normal distance from origin
- A, B, C, D = coefficients

- $\alpha,\ \beta,\ \gamma$ = normal direction angles

- $\cos\alpha,\ \cos\beta,\ \cos\gamma$ = normal direction cosines

- $l,\ m,\ n$ = normal direction numbers

• **n** = normal vector

= any nonzero vector normal to the plane

eg

$(\cos\alpha, \ \cos\beta, \ \cos\gamma)$ or $(l, \ m, \ n)$

• **A** = (A, B, C) = coordinate coefficient vector

Δ standard scalar forms

of the canonical rectangular equations of the plane

• general form

$$Ax + By + Cz + D = 0$$

• intercept form

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

• three - point determinant form

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0$$

• normal form

$$x\cos\alpha + y\cos\beta + z\cos\gamma - p = 0$$

• point - normal - direction - cosines form

$$(x - x_0)\cos\alpha + (y - y_0)\cos\beta + (z - z_0)\cos\gamma = 0$$

• point - normal - direction - numbers form

$$(x - x_0)\,l + (y - y_0)\,m + (z - z_0)\,n = 0$$

• parametric form

$$\begin{cases} x = x_0 + l'\,t + l''\,s \\ y = y_0 + m'\,t + m''\,s \\ z = z_0 + n'\,t + n''\,s \end{cases}$$

wh (l', m', n') & (l'', m'', n'')

are linearly independent vectors

that are parallel to the plane

or equivalently

normal to the normal vector

Δ standard vector forms
of the canonical rectangular equations of the plane

• general form

$$\mathbf{A} \cdot \mathbf{r} + \mathrm{D} \ = \ 0$$

• first three - point form

$$(\mathbf{r} - \mathbf{r}_1) \cdot (\mathbf{r}_2 - \mathbf{r}_1) \times (\mathbf{r}_3 - \mathbf{r}_1) \ = \ 0$$

• second three - point form

$$\left| \mathbf{r} - \mathbf{r}_1, \ \mathbf{r}_2 - \mathbf{r}_1, \ \mathbf{r}_3 - \mathbf{r}_1 \right| \ = \ 0$$

• point - normal form

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$$

• parametric form

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{l}' t + \mathbf{l}'' s$$

wh $\mathbf{l}'$ & $\mathbf{l}''$

are linearly independent vectors

that are parallel to the plane

or equivalently

normal to the normal vector

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• particular points on line

$P_0(x_0, y_0, z_0)$,

$P_1(x_1, y_1, z_1)$,

$P_2(x_2, y_2, z_2)$

• coordinate vector variable

$= \mathbf{r} = (x, y, z)$

• particular points on line

$\mathbf{r}_0 = (x_0, y_0, z_0)$,

$\mathbf{r}_1 = (x_1, y_1, z_1)$,

$\mathbf{r}_2 = (x_2, y_2, z_2)$

- A, B, C, D, E, F,

A_1, B_1, C_1, D_1,

A_2, B_2, C_2, D_2

= coefficients

- α, β, γ = direction angles

- $\cos\alpha, \cos\beta, \cos\gamma$ = direction cosines

- l, m, n = direction numbers

- $\mathbf{l}$ = direction vector

= any nonzero vector parallel to the line

eg

$(\cos\alpha,\ \cos\beta,\ \cos\gamma)$ or (l, m, n)

- $\mathbf{A}_1 = (A_1, B_1, C_1)$,

  $\mathbf{A}_2 = (A_2, B_2, C_2)$

= coordinate coefficient vectors

Δ standard scalar forms

of the canonical rectangular equations of the line

• two - plane form

$$\begin{cases} A_1 x + B_1 y + C_1 z + D_1 = 0 \\ A_2 x + B_2 y + C_2 z + D_2 = 0 \end{cases}$$

• first projection - plane form

$$A x + D = B y + E = C z + F$$

• second projection - plane form

$$\begin{cases} A x - B y + D - E = 0 \\ B y - C z + E - F = 0 \\ C z - A x + F - D = 0 \end{cases}$$

• two - point form

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

• point - direction - cosines form

$$\frac{x - x_0}{\cos\alpha} = \frac{y - y_0}{\cos\beta} = \frac{z - z_0}{\cos\gamma}$$

• point - direction - numbers form

$$\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$$

• parametric form

$$\begin{cases} x = x_0 + l\,t \\ y = y_0 + m\,t \\ z = z_0 + n\,t \end{cases}$$

Δ standard vector forms

of the canonical rectangular equations of the line

• two - plane form

$$\begin{cases} \mathbf{A}_1 \cdot \mathbf{r} + D_1 = 0 \\ \mathbf{A}_2 \cdot \mathbf{r} + D_2 = 0 \end{cases}$$

• two - point form

$$(\mathbf{r} - \mathbf{r}_1) \times (\mathbf{r}_2 - \mathbf{r}_1) = \mathbf{0}$$

• point - direction form

$$(\mathbf{r} - \mathbf{r}_0) \times \mathbf{l} = \mathbf{0}$$

• parametric form

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{l}\, t$$