A Semi-Systematic Sampling of Trig Identities: Part III

#25 of Gottschalk's Gestalts

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GG25-2

Δ fcn(A + B) ito fcn & cfcn of A & B

- $\sin(A+B) = \sin A \cos B + \cos A \sin B$

- $\cos(A+B) = \cos A \cos B - \sin A \sin B$

- $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B} = \dfrac{\cot A + \cot B}{\cot A \cot B - 1}$

- $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot A + \cot B} = \dfrac{1 - \tan A \tan B}{\tan A + \tan B}$

- $\sec(A+B) = \dfrac{\sec A \sec B \csc A \csc B}{\csc A \csc B - \sec A \sec B}$

- $\csc(A+B) = \dfrac{\sec A \sec B \csc A \csc B}{\sec A \csc B + \csc A \sec B}$

Δ fcn(A – B) ito fcn & cfcn of A & B

- $\sin(A - B) = \sin A \cos B - \cos A \sin B$

- $\cos(A - B) = \cos A \cos B + \sin A \sin B$

- $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B} = \dfrac{\cot B - \cot A}{\cot A \cot B + 1}$

- $\cot(A - B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A} = \dfrac{1 + \tan A \tan B}{\tan A - \tan B}$

- $\sec(A - B) = \dfrac{\sec A \sec B \csc A \csc B}{\csc A \csc B + \sec A \sec B}$

- $\csc(A - B) = \dfrac{\sec A \sec B \csc A \csc B}{\sec A \csc B - \csc A \sec B}$

Δ product of sin & cos of 2 angles as sum

- $2\sin A \sin B \quad = \quad -\cos(A+B)+\cos(A-B)$
- $2\sin A \cos B \quad = \quad \sin(A+B)+\sin(A-B)$
- $2\cos A \sin B \quad = \quad \sin(A+B)-\sin(A-B)$
- $2\cos A \cos B \quad = \quad \cos(A+B)+\cos(A-B)$

Δ product of tan & cot of 2 angles as quotient

- $$\tan A \tan B = \frac{\tan A + \tan B}{\cot A + \cot B} = -\frac{\tan A - \tan B}{\cot A - \cot B} = \frac{\cos(A-B) - \cos(A+B)}{\cos(A-B) + \cos(A+B)}$$

- $$\tan A \cot B = \frac{\tan A + \cot B}{\cot A + \tan B} = -\frac{\tan A - \cot B}{\cot A - \tan B} = \frac{\sin(A+B) + \sin(A-B)}{\sin(A+B) - \sin(A-B)}$$

- $$\cot A \tan B = \frac{\cot A + \tan B}{\tan A + \cot B} = -\frac{\cot A - \tan B}{\tan A - \cot B} = \frac{\sin(A+B) - \sin(A-B)}{\sin(A+B) + \sin(A-B)}$$

- $$\cot A \cot B = \frac{\cot A + \cot B}{\tan A + \tan B} = -\frac{\cot A - \cot B}{\tan A - \tan B} = \frac{\cos(A-B) + \cos(A+B)}{\cos(A-B) - \cos(A+B)}$$

Δ sum & difference of sin or cos of 2 angles as product

- $\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

- $\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$

- $\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

- $\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$

Δ sum & difference of tan & cot of 2 angles as quotient

- $\tan A + \tan B = \dfrac{\sin(A+B)}{\cos A \cos B}$

- $\tan A + \cot B = \dfrac{\cos(A-B)}{\cos A \sin B}$

- $\cot A + \tan B = \dfrac{\cos(A-B)}{\sin A \cos B}$

- $\cot A + \cot B = \dfrac{\sin(A+B)}{\sin A \sin B}$

- $\tan A - \tan B = \dfrac{\sin(A - B)}{\cos A \cos B}$

- $\tan A - \cot B = -\dfrac{\cos(A + B)}{\cos A \sin B}$

- $\cot A - \tan B = \dfrac{\cos(A + B)}{\sin A \cos B}$

- $\cot A - \cot B = -\dfrac{\sin(A - B)}{\sin A \sin B}$

Δ sum & difference of sec or csc of 2 angles as product

- $\sec A + \sec B = 2 \sec A \sec B \cos \frac{A+B}{2} \cos \frac{A-B}{2}$

- $\sec A - \sec B = 2 \sec A \sec B \sin \frac{A+B}{2} \sin \frac{A-B}{2}$

- $\csc A + \csc B = 2 \csc A \csc B \sin \frac{A+B}{2} \cos \frac{A-B}{2}$

- $\csc A - \csc B = -2 \csc A \csc B \cos \frac{A+B}{2} \sin \frac{A-B}{2}$

Δ difference of squares of sin & cos of 2 angles

- $\sin^2 A - \sin^2 B$

$= (\sin A + \sin B)(\sin A - \sin B)$

$= \cos^2 B - \cos^2 A$

$= (\cos B + \cos A)(\cos B - \cos A)$

$= \sin(A + B)\sin(A - B)$

- $\sin^2 A - \cos^2 B$

$= (\sin A + \cos B)(\sin A - \cos B)$

$= \sin^2 B - \cos^2 A$

$= (\sin B + \cos A)(\sin B - \cos A)$

$= -\cos(A + B)\cos(A - B)$

- $\cos^2 A - \sin^2 B$

$= (\cos A + \sin B)(\cos A - \sin B)$

$= \cos^2 B - \sin^2 A$

$= (\cos B + \sin A)(\cos B - \sin A)$

$= \cos(A + B)\cos(A - B)$

- $\cos^2 A - \cos^2 B$

$= (\cos A + \cos B)(\cos A - \cos B)$

$= \sin^2 B - \sin^2 A$

$= (\sin B + \sin A)(\sin B - \sin A)$

$= -\sin(A + B)\sin(A - B)$

Δ quotients of sum & difference of sin & cos of 2 angles

- $$\frac{\sin A - \sin B}{\sin A + \sin B} = \cot \frac{A+B}{2} \tan \frac{A-B}{2}$$

- $$\frac{\cos A + \cos B}{\sin A + \sin B} = \cot \frac{A+B}{2}$$

- $$\frac{\cos A - \cos B}{\sin A + \sin B} = -\tan \frac{A-B}{2}$$

- $$\frac{\sin A + \sin B}{\sin A - \sin B} = \tan\frac{A+B}{2}\cot\frac{A-B}{2}$$

- $$\frac{\cos A + \cos B}{\sin A - \sin B} = \cot\frac{A-B}{2}$$

- $$\frac{\cos A - \cos B}{\sin A - \sin B} = -\tan\frac{A+B}{2}$$

- $$\frac{\sin A + \sin B}{\cos A + \cos B} = \tan \frac{A + B}{2}$$

- $$\frac{\sin A - \sin B}{\cos A + \cos B} = \tan \frac{A - B}{2}$$

- $$\frac{\cos A - \cos B}{\cos A + \cos B} = -\tan \frac{A + B}{2} \tan \frac{A - B}{2}$$

- $$\frac{\sin A + \sin B}{\cos A - \cos B} = -\cot\frac{A - B}{2}$$

- $$\frac{\sin A - \sin B}{\cos A - \cos B} = -\cot\frac{A + B}{2}$$

- $$\frac{\cos A + \cos B}{\cos A - \cos B} = -\cot\frac{A + B}{2}\cot\frac{A - B}{2}$$

Δ quotients of sums & differences of tan & cot of 2 angles

- $\dfrac{\tan A + \cot B}{\tan A + \tan B} = \cot B \dfrac{\cos(A - B)}{\sin(A + B)}$

- $\dfrac{\cot A + \tan B}{\tan A + \tan B} = \cot A \dfrac{\cos(A - B)}{\sin(A + B)}$

- $\dfrac{\cot A + \cot B}{\tan A + \tan B} = \cot A \cot B$

- $\dfrac{\tan A - \tan B}{\tan A + \tan B} = \dfrac{\sin(A - B)}{\sin(A + B)}$

- $\dfrac{\tan A - \cot B}{\tan A + \tan B} = -\cot B \cot(A + B)$

- $\dfrac{\cot A - \tan B}{\tan A + \tan B} = \cot A \cot(A + B)$

- $\dfrac{\cot A - \cot B}{\tan A + \tan B} = -\cot A \cot B \dfrac{\sin(A - B)}{\sin(A + B)}$

- $$\frac{\tan A + \tan B}{\tan A + \cot B} = \tan B \frac{\sin(A+B)}{\cos(A-B)}$$

- $$\frac{\cot A + \tan B}{\tan A + \cot B} = \cot A \tan B$$

- $$\frac{\cot A + \cot B}{\tan A + \cot B} = \cot A \frac{\sin(A+B)}{\cos(A-B)}$$

- $$\frac{\tan A - \tan B}{\tan A + \cot B} = \tan B \tan(A-B)$$

- $$\frac{\tan A - \cot B}{\tan A + \cot B} = -\frac{\cos(A+B)}{\cos(A-B)}$$

- $$\frac{\cot A - \tan B}{\tan A + \cot B} = \cot A \tan B \frac{\cos(A+B)}{\cos(A-B)}$$

- $$\frac{\cot A - \cot B}{\tan A + \cot B} = -\cot A \tan(A-B)$$

- $$\frac{\tan A + \tan B}{\cot A + \tan B} = \tan A \frac{\sin(A+B)}{\cos(A-B)}$$

- $$\frac{\tan A + \cot B}{\cot A + \tan B} = \tan A \cot B$$

- $$\frac{\cot A + \cot B}{\cot A + \tan B} = \cot B \frac{\sin(A+B)}{\cos(A-B)}$$

- $$\frac{\tan A - \tan B}{\cot A + \tan B} = \tan A \tan(A-B)$$

- $$\frac{\tan A - \cot B}{\cot A + \tan B} = -\tan A \cot B \frac{\cos(A+B)}{\cos(A-B)}$$

- $$\frac{\cot A - \tan B}{\cot A + \tan B} = \frac{\cos(A+B)}{\cos(A-B)}$$

- $$\frac{\cot A - \cot B}{\cot A + \tan B} = -\cot B \tan(A-B)$$

- $$\frac{\tan A + \tan B}{\cot A + \cot B} = \tan A \tan B$$

- $$\frac{\tan A + \cot B}{\cot A + \cot B} = \tan A \frac{\cos(A - B)}{\sin(A + B)}$$

- $$\frac{\cot A + \tan B}{\cot A + \cot B} = \tan B \frac{\cos(A - B)}{\sin(A + B)}$$

- $$\frac{\tan A - \tan B}{\cot A + \cot B} = \tan A \tan B \frac{\sin(A - B)}{\sin(A + B)}$$

- $$\frac{\tan A - \cot B}{\cot A + \cot B} = -\tan A \cot(A + B)$$

- $$\frac{\cot A - \tan B}{\cot A + \cot B} = \tan B \cot(A + B)$$

- $$\frac{\cot A - \cot B}{\cot A + \cot B} = -\frac{\sin(A - B)}{\sin(A + B)}$$

- $$\frac{\tan A + \tan B}{\tan A - \tan B} = \frac{\sin(A+B)}{\sin(A-B)}$$

- $$\frac{\tan A + \cot B}{\tan A - \tan B} = \cot B \cot(A-B)$$

- $$\frac{\cot A + \tan B}{\tan A - \tan B} = \cot A \cot(A-B)$$

- $$\frac{\cot A + \cot B}{\tan A - \tan B} = \cot A \cot B \frac{\sin(A+B)}{\sin(A-B)}$$

- $$\frac{\tan A - \cot B}{\tan A - \tan B} = -\cot B \frac{\cos(A+B)}{\sin(A-B)}$$

- $$\frac{\cot A - \tan B}{\tan A - \tan B} = \cot A \frac{\cos(A+B)}{\sin(A-B)}$$

- $$\frac{\cot A - \cot B}{\tan A - \tan B} = -\cot A \cot B$$

- $$\frac{\tan A + \tan B}{\tan A - \cot B} = -\tan B \tan(A+B)$$

- $$\frac{\tan A + \cot B}{\tan A - \cot B} = -\frac{\cos(A-B)}{\cos(A+B)}$$

- $$\frac{\cot A + \tan B}{\tan A - \cot B} = -\cot A \tan B \frac{\cos(A-B)}{\cos(A+B)}$$

- $$\frac{\cot A + \cot B}{\tan A - \cot B} = -\cot A \tan(A+B)$$

- $$\frac{\tan A - \tan B}{\tan A - \cot B} = -\tan B \frac{\sin(A-B)}{\cos(A+B)}$$

- $$\frac{\cot A - \tan B}{\tan A - \cot B} = -\cot A \tan B$$

- $$\frac{\cot A - \cot B}{\tan A - \cot B} = \cot A \frac{\sin(A-B)}{\cos(A+B)}$$

- $$\frac{\tan A + \tan B}{\cot A - \tan B} = \tan A \tan (A + B)$$

- $$\frac{\tan A + \cot B}{\cot A - \tan B} = \tan A \cot B \frac{\cos (A - B)}{\cos (A + B)}$$

- $$\frac{\cot A + \tan B}{\cot A - \tan B} = \frac{\cos (A - B)}{\cos (A + B)}$$

- $$\frac{\cot A + \cot B}{\cot A - \tan B} = \cot B \tan (A + B)$$

- $$\frac{\tan A - \tan B}{\cot A - \tan B} = \tan A \frac{\sin (A - B)}{\cos (A + B)}$$

- $$\frac{\tan A - \cot B}{\cot A - \tan B} = -\tan A \cot B$$

- $$\frac{\cot A - \cot B}{\cot A - \tan B} = -\cot B \frac{\sin (A - B)}{\cos (A + B)}$$

- $$\frac{\tan A + \tan B}{\cot A - \cot B} = -\tan A \tan B \frac{\sin(A+B)}{\sin(A-B)}$$

- $$\frac{\tan A + \cot B}{\cot A - \cot B} = -\tan A \cot(A-B)$$

- $$\frac{\cot A + \tan B}{\cot A - \cot B} = -\tan B \cot(A-B)$$

- $$\frac{\cot A + \cot B}{\cot A - \cot B} = -\frac{\sin(A+B)}{\sin(A-B)}$$

- $$\frac{\tan A - \tan B}{\cot A - \cot B} = -\tan A \tan B$$

- $$\frac{\tan A - \cot B}{\cot A - \cot B} = \tan A \frac{\cos(A+B)}{\sin(A-B)}$$

- $$\frac{\cot A - \tan B}{\cot A - \cot B} = -\tan B \frac{\cos(A+B)}{\sin(A-B)}$$

Δ quotient of sum & difference of sec & csc of 2 angles

- $$\frac{\sec A - \sec B}{\sec A + \sec B} = \tan\frac{A+B}{2}\tan\frac{A-B}{2}$$

- $$\frac{\csc A + \csc B}{\sec A + \sec B} = \cot A \cot B \tan\frac{A+B}{2}$$

- $$\frac{\csc A - \csc B}{\sec A + \sec B} = -\cot A \cot B \tan\frac{A-B}{2}$$

- $$\frac{\sec A + \sec B}{\sec A - \sec B} = \cot\frac{A+B}{2}\cot\frac{A-B}{2}$$

- $$\frac{\csc A + \csc B}{\sec A - \sec B} = \cot A \cot B \cot\frac{A-B}{2}$$

- $$\frac{\csc A - \csc B}{\sec A - \sec B} = -\cot A \cot B \cot\frac{A+B}{2}$$

- $\dfrac{\sec A + \sec B}{\csc A + \csc B} = \tan A \tan B \cot \dfrac{A + B}{2}$

- $\dfrac{\sec A - \sec B}{\csc A + \csc B} = \tan A \tan B \tan \dfrac{A - B}{2}$

- $\dfrac{\csc A - \csc B}{\csc A + \csc B} = -\cot \dfrac{A + B}{2} \tan \dfrac{A - B}{2}$

- $$\frac{\sec A + \sec B}{\csc A - \csc B} = -\tan A \tan B \cot \frac{A - B}{2}$$

- $$\frac{\sec A - \sec B}{\csc A - \csc B} = -\tan A \tan B \tan \frac{A + B}{2}$$

- $$\frac{\csc A + \csc B}{\csc A - \csc B} = -\tan \frac{A + B}{2} \cot \frac{A - B}{2}$$

Δ some two - angle identities

- $\sin A + \sin B + \sin(A+B)$

$$= 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{A+B}{2}$$

- $\cos A + \cos B + \cos(A+B)$

$$= 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{A+B}{2} - 1$$

- $$\frac{\sin(A-B) + \sin A + \sin(A+B)}{\cos(A-B) + \cos A + \cos(A+B)} = \frac{\sin A}{\cos A} = \tan A$$

- $$\frac{\cos(A-B) + \cos A + \cos(A+B)}{\sin(A-B) + \sin A + \sin(A+B)} = \frac{\cos A}{\sin A} = \cot A$$

(note that $A-B$, A, $A+B$ are in arithmetic progression;
iow A in the arithmetic mean of $A+B$ and $A-B$)

- $\sin^2(A+B)$

$= \sin^2 A + \sin^2 B + 2\sin A \sin B \cos(A+B)$

- $\cos^2(A+B)$

$= \cos^2 A + \cos^2 B - 2\sin A \sin B \cos(A+B) - 1$