The 44 Most Serviceable Trig Identities

#22 of Gottschalk's Gestalts

A Series Illustrating Innovative Forms
of the Organization & Exposition
of Mathematics
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□ the 44 most serviceable trig identities
= the trig identities worth remembering
in symbol & in word

Δ the 3 product identities

Δ the 6 reciprocal identities

Δ the 2 quotient identities

Δ the 3 pythagorean identities

Δ the 3 addition formulas/identities

Δ the 3 subtraction formulas/identities

Δ the 3 double-angle formulas/identities

Δ the 3 half-angle formulas/identities

Δ the 6 cofunction-complement identities

Δ the 6 parity identities

Δ the 6 period identities

Δ the 3 product identities

$$\sin A \csc A = 1$$

$$\cos A \sec A = 1$$

$$\tan A \cot A = 1$$

in words

sine times cosecant equals one

cosine times secant equals one

tangent times cotangent equals one

note:
sine & cosecant
cosine & secant
tangent & cotangent
are reciprocal pairs

Δ the 6 reciprocal identities

$$\frac{1}{\sin A} = \csc A$$

$$\frac{1}{\cos A} = \sec A$$

$$\frac{1}{\tan A} = \cot A$$

$$\frac{1}{\cot A} = \tan A$$

$$\frac{1}{\sec A} = \cos A$$

$$\frac{1}{\csc A} = \sin A$$

in words

reciprocal of sine equals cosecant

reciprocal of cosine equals secant

reciprocal of tangent equals cotangent

reciprocal of cotangent equals tangent

reciprocal of secant equals cosine

reciprocal of cosecant equals sine

Δ the 2 quotient identities
for tangent & cotangent

$$\tan A = \frac{\sin A}{\cos A}$$

$$\cot A = \frac{\cos A}{\sin A}$$

in words

tangent equals sine over cosine

cotangent equals cosine over sine

∆ the 3 pythagorean identities

$$\sin^2 A + \cos^2 A = 1$$

$$1 + \tan^2 A = \sec^2 A$$

$$1 + \cot^2 A = \csc^2 A$$

in words

sine square plus cosine square equals one

one plus tangent square equals secant square

one plus cotangent square equals cosecant square

Δ the 3 addition formulas/identities
for sine, cosine, tangent

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

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in words

sine of sum of two angles
equals
sine of first times cosine of second
plus
cosine of first times sine of second

cosine of sum of two angles
equals
cosine of first times cosine of second
minus
sine of first times sine of second

tangent of sum of two angles
equals
sum of tangents of angles
over
one minus their product

Δ the 3 subtraction formulas/identities
for sine, cosine, tangent

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

in words

sine of difference of two angles
equals
sine of first times cosine of second
minus
cosine of first times sine of second

cosine of difference of two angles
equals
cosine of first times cosine of second
plus
sine of first times sine of second

tangent of difference of two angles
equals
difference of tangents of angles
over
one plus their product

Δ the 3 double-angle formulas/identities
for the sine, cosine, tangent

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

in words

sine of double angle
equals
two times sine of angle times cosine of angle

cosine of double angle
equals
cosine square of angle
minus
sine square of angle

tangent of double angle
equals
two times tangent of angle
over
one minus tangent square of angle

Δ the 3 half-angle formulas/identities
for sine, cosine, tangent

$$\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}}$$

$$\cos\frac{A}{2} = \sqrt{\frac{1+\cos A}{2}}$$

$$\tan\frac{A}{2} = \sqrt{\frac{1-\cos A}{1+\cos A}} = \frac{1-\cos A}{\sin A} = \frac{\sin A}{1+\cos A}$$

in words

sine of half angle
equals
square root of
one minus cosine of angle over two

cosine of half angle
equals
square root of
one plus cosine of angle over two

tangent of half angle
equals
square root of
one minus cosine of angle
over
one plus cosine of angle
equals
one minus cosine of angle over sine of angle
equals
sine of angle over one plus cosine of angle

Δ the 6 cofunction-complement identities

$$\sin A = \cos(90^{o} - A)$$

$$\cos A = \sin(90^{o} - A)$$

$$\tan A = \cot(90^{o} - A)$$

$$\cot A = \tan(90^{o} - A)$$

$$\sec A = \csc(90^{o} - A)$$

$$\csc A = \sec(90^{o} - A)$$

in words

function of angle
equals
cofunction of complement of angle

& equivalently

function of complement of angle
equals
cofunction of angle

Δ the 6 parity identities

$$\sin(-A) = -\sin A$$

$$\cos(-A) = \cos A$$

$$\tan(-A) = -\tan A$$

$$\cot(-A) = -\cot A$$

$$\sec(-A) = \sec A$$

$$\csc(-A) = -\csc A$$

in words

sine, tangent, cotangent, cosecant
are
odd functions

&

cosine, secant
are
even functions

Δ the 6 period identities

$$\sin(A + 360^o) = \sin A$$

$$\cos(A + 360^o) = \cos A$$

$$\tan(A + 180^o) = \tan A$$

$$\cot(A + 180^o) = \cot A$$

$$\sec(A + 360^o) = \sec A$$

$$\csc(A + 360^o) = \csc A$$

in words

all six basic trig functions
are periodic

&

sine, cosine, secant, cosecant
are periodic functions
with period
360 degrees = 2π radians

&

tangent, cotangent
are periodic functions
with period
180 degrees = π radians

Δ other trig identities
such as

the 4 sum-to-product formulas/identities

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

&

the 4 product-to-sum formulas/identities

$$2 \sin A \sin B = -\cos(A+B) + \cos(A-B)$$

$$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

may be retrieved from a formulary or derived quickly
but are not worth remembering;
remember only their existence