Sets of Integers

#15 of Gottschalk's Gestalts

A Series Illustrating Innovative Forms
of the Organization & Exposition
of Mathematics
by Walter Gottschalk

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GG15-2

- the set of all integers

= the set of integers

= the integer set

= the integers

= $\{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$

= dn $\mathbb{Z}$

= rd cap open zee = zee

- the set of all nonnegative integers

= the set of nonnegative integers

= the nonnegative integer set

= the nonnegative integers

= $\{0,\ 1,\ 2,\ 3,\ \cdots\}$

= dn $\mathbb{N}$

= rd cap open en = en

- the set of all positive integers

= the set of positive integers

= the positive integer set

= the positive integers

= $\{1,\ 2,\ 3,\ \cdots\}$

= dn $\mathbb{P}$

= rd cap open pe = pe

- the set of all nonpositive integers

= the set of nonpositive integers

= the nonpositive integer set

= the nonpositive integers

= $\{0, \ -1, \ -2, \ -3, \cdots\}$

= dn $\overline{\mathbb{N}}$

= rd cap open en bar = en bar

- the set of all negative integers

= the set of negative integers

= the negative integer set

= the negative integers

= $\{-1, \ -2, \ -3, \cdots\}$

= dn $\overline{\mathbb{P}}$

= rd cap open pe bar = pe bar

• note that the ' minus sign' in $\overline{\mathbb{N}}$ and $\overline{\mathbb{P}}$ are
conveniently placed above the letters and not before;
as for the ' etymology' of the capital open letters,
$\mathbb{Z}$ is from die Zahl (German) = number
$\mathbb{N}$ is from nonnegative, natural, number
$\mathbb{P}$ is from positive;
the open type style,
which is a style seldom appearing in mathematical literature,
was adopted for the sake of this particular usage
and for immediate recognition;
these letters are now in almost universal use
with the present meaning

• an alternative notation based on $\mathbb{Z}$

the nonnegative integer set $= \mathbb{N} = \mathbb{Z}_{+}$

the nonpositive integer set $= \overline{\mathbb{N}} = \mathbb{Z}_{-}$

the positive integer set $= \mathbb{P} = \mathbb{Z}_{+0}$

the negative integer set $= \overline{\mathbb{P}} = \mathbb{Z}_{-0}$

- the set of all nonzero integers

= the set of nonzero integers

= the nonzero integer set

= the nonzero integers

= $\{\cdots, -3, -2, -1, 1, 2, 3, \cdots\}$

= dn $\mathbb{Z}_*$

= rd $\mathbb{Z}$ (sub) star

think of $\mathbb{Z}_*$ as ' punctured $\mathbb{Z}$'
where zero is punched out of $\mathbb{Z}$,
leaving a hole represented by the star

- the set of all even integers / numbers

= the set of even integers / numbers

= the even integer / number set

= the even integers / numbers

= $\{\cdots, -6, -4, -2, 0, 2, 4, 6, \cdots\}$

= $\{2n \mid n \in \mathbb{Z}\}$

= $2\mathbb{Z}$

- the set of all odd integers / numbers

= the set of odd integers / numbers

= the odd integer / number set

= the odd integers / numbers

= $\{\cdots, -5, -3, -1, 1, 3, 5, \cdots\}$

= $\{2n + 1 \mid n \in \mathbb{Z}\}$

= $2\mathbb{Z} + 1$

- the set of all integer multiples of n wh n $\in$ pos int

= the set of integer multiples of n

= the integer multiple set of n

= the integer multiples of n

= $\{\cdots, -3n, -2n, -n, 0, n, 2n, 3n, \cdots\}$

= $\{kn \mid k \in \mathbb{Z}\}$

= $n\mathbb{Z}$

$n \in$ pos int $\Rightarrow$

- the set of all equivalence classes of integers modulo n

= the set of equivalence classes of integers mod n

= the set of integer equivalence classes mod n

= the quotient set of integers mod n

= the integer quotient set mod n

= $\{n\mathbb{Z} + k \mid k \in \mathbb{Z}[0,\ n-1]\}$

= $\mathbb{Z} / n\mathbb{Z}$

= dn $\mathbb{Z}_n$

• intervals of integers

every set of integers has a natural order
and hence intervals of various kinds
are automatically defined for sets of integers;
here are three examples
of blocks and rays
in the entire integer spread $\mathbb{Z}$

• blocks of integers

$a,\ b \in \mathbb{Z}\ \&\ a \le b \Rightarrow$

the block of integers from a to b

= the integer block from a to b

= the set of all integers from a to b

= the closed interval of $\mathbb{Z}$ from a to b

$= \{a,\ a+1,\ a+2, \cdots,\ b\}$

$= \{n \mid n \in \mathbb{Z}\ \&\ a \le n \le b\}$

$= \mathbb{Z}[a,\ b]$

• right rays of integers

$a \in \mathbb{Z} \Rightarrow$

the right ray of integers from a

= the integer right ray from a

= the set of all integers from a to the right

= the right ray of $\mathbb{Z}$ from a

$= \{a,\ a+1,\ a+2,\ \cdots\}$

$= \{n \mid n \in \mathbb{Z}\ \&\ a \le n\}$

$= \mathbb{Z}[a, \rightarrow)$

• left rays of integers

$a \in \mathbb{Z} \Rightarrow$

the left ray of integers to a

= the integer left ray to a

= the set of all integers to a from the left

= the left ray of $\mathbb{Z}$ to a

= $\{\cdots, a-2, a-1, a\}$

= $\{n \mid n \in \mathbb{Z} \ \& \ n \leq a\}$

= $\mathbb{Z}(\leftarrow, a]$

- the file of length n wh n $\in$ pos int

= the n - file

= dn $\underline{n}$

= rd n file

= df the set of the first n positive integers

= $\{1,\ 2,\ 3,\ \cdots, n\}$

= $\mathbb{P}[1,\ n]$

- the set of all prime integers / numbers

= the set of prime integers / numbers

= the prime integer / number set

= the prime integers / numbers

= the set of all primes

= the set of primes

= the prime set

= the primes

= $\{2,\ 3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ \cdots\}$

= dn Prm = Pr = P

- the set of all odd prime integers / numbers

= the set of odd prime integers / numbers

= the odd prime integer / number set

= the odd prime integers / numbers

= the set of all odd primes

= the set of odd primes

= the odd prime set

= the odd primes

= $\{3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ \cdots\}$

= dn P_*

= rd P (sub) star

- the set of all composite integers / numbers

= the set of composite integers / numbers

= the composite integer / number set

= the composite integers / numbers

= the set of all composites

= the set of composites

= the composite set

= the composites

= $\{1,\ 4,\ 6,\ 8,\ 9,\ 10,\ 12,\ 14,\ \cdots\}$

= dn Cmp = Cm = C

- the set of all plural composite integers / numbers

= the set of plural composite integers / numbers

= the plural composite integer / number set

= the plural composite integers / numbers

= the set of all plural composites

= the set of plural composites

= the plural composite set

= the plural composites

= $\{4,\ 6,\ 8,\ 9,\ 10,\ 12,\ 14,\ 15,\ \cdots\}$

= dn C_*

= rd C (sub) star

- the sequence of all prime integers / numbers

= the sequence of prime integers / numbers

= the prime integer / number sequence

= the sequence of all primes

= the sequence of primes

= the prime sequence

= $(2,\ 3,\ 5,\ 7,\ 11,\ 13,\ 17,\ 19,\ \cdots)$

= $\mathrm{dn}\ (p_1,\ p_2,\ p_3,\ \cdots)$

= $\mathrm{dn}\ \overline{\mathrm{P}}\mathrm{rm}\ =\ \overline{\mathrm{P}}\mathrm{r}\ =\ \overline{\mathrm{P}}$

- the sequence of all composite integers / numbers

= the sequence of composite integers / numbers

= the composite integer / number sequence

= the sequence of all composites

= the sequence of composites

= the composite sequence

= $(1,\ 4,\ 6,\ 8,\ 9,\ 10,\ 12,\ 14,\ \cdots)$

= $\text{dn}\ (c_1,\ c_2,\ c_3, \cdots)$

= $\text{dn}\ \overline{\text{C}}\text{mp}\ =\ \overline{\text{C}}\text{m}\ =\ \overline{\text{C}}$

note: general terminological & notational compression

- the set of all items
= the set of items
= the item set
= the items

if the set of all items
is provided with a structure
to obtain a system
(note the triple ess dictum
system = set + structure)
then

- the system of all items
= the system of items
= the item system
and
the same symbol may be used
for both set & system

eg

• the set of all integers

= the set of integers

= the integer set

= the integers

= $\mathbb{Z}$

&

• the ring of all integers

= the ring of integers

= the integer ring

= $\mathbb{Z}$

&

• the ring of all Gaussian integers

= the ring of Gaussian integers

= the Gaussian integer ring

= df $\mathbb{Z} + i\mathbb{Z}$

= $\mathbb{Z}[i]$