My Style

#8 of Gottschalk's Gestalts

A Series Illustrating Innovative Forms
of the Organization & Exposition
of Mathematics
by Walter Gottschalk

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☐ My Style

This packet sets forth in some detail the principles and practices governing the present series of expositions of mathematics. This packet uses the traditional format mostly altho it describes nontraditional formats.

I have learned/studied/thought mathematics for seven & a half decades. I have taught mathematics to undergraduate and graduate students in courses that ranged from pre-freshman-level remedial courses thru dissertation-level courses for five decades. I have pursued and published mathematics research for four decades. I have supervised mathematics research for three decades. During the last two decades I have been retired from teaching duties but I have been busily engaged in thinking, reading, and writing mathematics. As a result of these experiences I have come upon certain ideas leading to nontraditional forms of the organization and exposition of mathematics. I have been incorporating these ideas in a series of laser-printed packets and I plan to continue to do so. But in addition to offering these exemplars over the Internet I want to say something very explicit about what these ideas are and to offer a rationale for them.

I think now of any combination of paper-and-pencil mathematcs and computer-assisted mathematics that leads to printed-on-paper mathematcs exposition.

The expression of mathematics must be embedded in a natural language since a completely formalized logic is not a practical alternative and mental telepathy is not yet an available alternative. I maintain that even a small fraction of mathematics embedded in a completely formalized logic, altho theoretically possible, would be inordinately long, strangely opaque, not widely known, and it is simply not the way the conscious mind operates.

I adopt English as the natural-language vehicle for mathematics because:

• in my opinion English is the best language for mathematics

• English is my native language and I have been thinking, speaking, hearing, reading, writing, learning, teaching mathematics in English for seven and a fraction decades

• English is the best candidate for an all-purpose world language; English is spoken around the world and has wider dispersion than any other language; English is also an official language of the United Nations

• it is estimated that there are 800 million speakers of English in the world today, this number of speakers of a single language being exceeded only by the one billion speakers of Mandarin (who are mostly in China)

• English is not only the recognized international language of science but English is also recognized as the world-wide language for business, communication, computers and the Internet, diplomacy, flight, medicine, sports. technology, tourism

• English is the biggest borrower of all time, absorbing words from all directions at a great rate

• English is the richest language of all time, that is, it has the largest vocabulary (something like a half-million words apprearing in a general standard unabridged dictionary plus at least a half-million words appearing in highly specialized, mostly scientific, usage) and it is the most expressive

• English has few inflections and depends heavily on word order for meaning; mathematics and logic depend crucially on the order of symbols for meaning (incidentally, I consider logic to be a branch of mathematics rather than the other way around)

• the basic grammar of English seems simple and relatively compatible with propositional and predicate logic

• English abbreviates well and mixes well with logical and mathematical symbolism

• English/Latin letters and their combinations contribute a large fraction of the notation of mathematics and logic

• English is written horizontally on lines from left to right and the lines are written from top to bottom, the way mathematics is generally written (there being notable exceptions such as some calculations, particular notations, subscripts, superscripts, diagrams, labels on diagrams, etc)

• the great bulk of the past/present mathematical literature was/is either originally in English or has been/is being translated into English

The customary sheet of working paper is white and rectangular of dimensions 8 & 1/2 by 11 inches. From long years of experience - my own and others' - I deem this color and size to be optimal not only for writing but also for printing. Reinforcement of this judgement comes from the ease and convenience of handling and storing this shape and size of paper, from its use for many graphs and charts and tables, from its use in business correspondence, from its frequent use in computer printers, and from its ubiquitous availability. However, variations in color and size and the use of cards instead of flexible paper are also to be recognized.

I judge black pencil and black type to be best. The contrast between white background and black marks seems to make for easy reading and steady looking. Also experience, usage, and availability can be cited in support of the choice of black for routine use. Only one side of the sheet is to be written/printed on, for reasons that will appear.

We now define the standard physical module for the exposition of mathematics: one unattached white rectangular 8 & 1/2 by 11 inch sheet of paper with black marks appearing only on one side.

This module permits ultimate flexibility in arranging sheets in combination for impact, unification, comparison, and contrast. Larger units of visual presentation can be constructed by juxtaposing several sheets in various patterns on a flat surface. Let us call this capability 'the gestalt generator'. No more flipping pages back and forth many times to match up things. A display of several sheets can be pinned on a bulletin board.

The study of particular arrangements of sheets should be a significant part of the reader's activity in addition to the study of individual sheets. Special arrangements of sheets may be suggested but let reader's choice flourish.

It seems possible that the 'gestalts' and 'tableaus' that occur in 'spreading the sheets' may in some small way imitate the action of the mind in its internal organization of knowledge.

The size of the type should in general be about that of handwriting. The exact size of the type will depend on the context. Usually I prefer Geneva regular 16 points when there is little if any symbolism in the text and Times regular 20 points when symbolism occurs with the word text. I regard the size of symbols, letters, words, etc as of paramount importance. Let loud = large & clear be the order of the day.

The position of words and symbols on the page and relative to one another is basic. They need lots of space between to breath and function. No crowding just ‘to save space’. Think rather of saving the time and assisting the mental effort of the reader.

There should be no typographic distinction between words and mathematical symbols because words are symbols too and part-and-parcel of the exposition which should be regarded as a unity. Indeed, I regard mathematical terminology as part of mathematical notation.

The sheets must remain unbound and loose so they can be easily shuffled around. A large paper clip can conveniently and temporarily hold up to 12 sheets or somewhat more. A small spring clip can conveniently and temporarily hold up to 25 sheets or somewhat more. A clear plastic envelope can hold one or more clipped bundles. The sheets on a particular topic coming off the laser printer, clipped together and placed in a clear plastic envelope, make the compete finished product.
What do I call the finished product? How about ‘packet’?
It has overtones of ‘pack it’ and ‘kit’.

As an experiment I reduced the individual sheets of a multiple-page gestalt to one-quarter size. The result was not as impressive and effective as the original. There seems to be something about the visual angle of words/symbols/formulas that makes for maximum impact. Handwriting size more or less for letters on paper? Half-finger size, say, for letters on the chalkboard when you are standing next to it? Of course, if storage of data by marks on paper is the aim, then smallness and compactness of marks is better and both sides of a sheet are to be used.

I am experimenting with the application of the following guidelines.

• always simplify and unify; ‘simplify’ may mean to break up into smaller units which may later be ‘unified’ into a larger unit;
of course, ‘simplify and unify’ may also mean a major or minor change in point of view

• always facilitate reading; always speed comprehension; always seek the revealing insight - these are excellent exhortations but operational details are hard to come by

• strive to write mathematics the way it ought to be:
beautiful & clear & flowing & inevitable & irresistable

• mathematics can be infinitely concentrated/condensed in the sense that larger and larger portions of mathematics can be recognized by the mind as units to be comprehended in shorter and shorter times; the exposition should reflect this very notable property of mathematics, saying more and more in less and less space-time; to try to summarize: mathematics can be compressed to potentially infinite density

• it seems that at comfortable reading distance a dozen
8 & 1/2 by 11 inch sheets arranged in three rows and four columns rather effectively fill up the visual field and
are easily scanned

• use fully the two-dimensional aspect of the page; speech is one-dimensional (variable = time) and the customary use of a page in writing/printing converts the two-dimensional page into a one-dimensional ribbon

• the arrangement of words and symbols on a page should help the eye in moving and the mind in understanding, not make it unduely necessary to ‘dig’

• place phrases and even individual words on separate lines to make for greater ease in reading and understanding by having the rhythms/units of the language match the lines of writing or print; for natural language this idea is nontraditional; for mathematical symbolism this idea is traditional

• abbreviate lots of words and phrases, generally without the use of periods

• make generous use of the equaters such as the definer and the denoter

• make some use of logical symbolism to replace some words and to say things briefly; it is paradoxical that to eliminate natural language entirely thru the use of logical symbolism leads to a dense and virtually incomprehensible exposition; this is a central problem in programming

• combine the classical notation and terminology of analysis with the modern notation and terminology of set theory and logic for the sake of greater precision and clarity

• make use of markers such as
open squares = boxes, triangles = trines, bullets = dots
which are usually at the left of the line

• use lowercase letters most of the time; lowercase printed letters are closer to cursive handwriting than capital printed letters, and words in lowercase printed letters are likely to be more easily recognized and read

• the section heading of a definition, theorem, remark, etc, should often, if not generally, have a brief suggestive announcement of its contents; eg the standard name of a theorem, it it has one

• use lots of charts, diagrams, graphs, line drawings, tables; these are highly visual devices that in principle can be comprehended at a glance or recalled at a glance once they have been studied

• use lots of geometry and pictures where possible and appropriate

• recognize the continual complementing/contrasting counterpoint between algebra and geometry

• in general, mix together the branches of mathematics

• make use of analogy, comparison, contrast, duality wherever possible

• form lots of gestalts and tableaus

• use arrows to indicate the flow of thought

• give generous hints of important topics and theorems that the student is likely to encounter in future courses and experience; anticipate a bit the mathematics the student will meet in the future; it is always nice to know where you are going and something of what you will see, if that is possible

• always point out and emphasize the elegant/esthetc aspect of mathematics

• make room for a little humor in mathematics which (to me) is natural and indeed inevitable; the best mathematical humor teaches as well as entertains; some humor mixed in with the mathematics may help the learning and remembering processes

• alliteration, rhyme, puns, and word play in general can help to frame and clarify ideas; poetry and prose of high literary quality can be used to good advantage

• say things precisely but also again imprecisely for the sake of brevity, capsule comprehension, suggestiveness, and memory retention

• make use of mnemonics

• use paraphrases in words and symbols, this being helpful to the understanding; a little redundancy in exposition is not a bad thing

• list synonyms systematically; I remember that in my student days I was continually surprised by the instances in which many words were used to mean the same thing or similar things altho they were never in my reading collected together in one place and the different shades of meaning, if any, explained

• list alternative notation/terminology; in particular, list choices in the use of prepositions

• point out deliberate ambiguities in notation and terminolgy; meaning is a function of custom and context

• give both the semantic and the syntactic names of symbols

• say how to write new symbols such as open-face capital letters

• say how to read symbols and formulas

• say how to pronounce words and names by the use of simple phonetic spelling

• use some simplified spelling such as ‘thru’ for ‘through’

• use some words and phrases of foreign languages in mathematics discourse; mathematics is a language too (among other things)

• follow the British practice of using single quotation marks rather than double quotation marks at first; the use of quotation marks in forming names should be minimized; they tend to clutter; I know of no mathematician who makes mistakes in mathematics because he/she does not know the difference between an object and a name of the object

• follow the Spanish practice of using the initial inverted question mark and exclamation mark as well as the terminal ones

• displayed mathematcal symbolism and punctuation arising from the structure of an English sentence do not mix well; the punctuation should be deleted

• use colors in words and diagrams for special effects and emphases

• the print display on a sheet of a packet may also serve at the same time for the textbook, the chalkboard, the projector, the computer screen; note that a panel of a chalkboard and a transparency for a projector and the screen of a computer cannot be turned over

• what the textbook has, what the teacher writes on the board, what the the students write down in their notebooks and on their homework and tests, should not be all that different; in traditional practice they are likely to be substantially different

• make generous comments of an artistic, biographical, esthetic, historical, humorous, linguistic, literary, philosophical, scientific nature to show that mathematics is in everything and is found everywhere

• ‘And now abideth grace, scope, clarity, these three; but the greatest of these is clarity.’ Anon. After I Cor 13:13 KJV.

The student that I am constantly asking to review my written exposition is myself when I was a student, from kindergarten to graduate school. What would I have liked to know? What would I have liked to read and study? Now I can correct the expository faults that my reading encountered then. I did not recognize expository faults. I always blamed myself rather than the text for any lack of understanding. But sometimes it was not my fault.

When a mathematics teacher lectures and writes at the chalkboard or besides an overhead projector, there is tremendous pressure to express as much as possible in as few written symbols as possible. What likely happens is that much is said but little is written down because long-hand writing cannot keep up with speech. It is often the case, if not almost always, that a student taking notes writes down just what he/she sees on the board/screen. I would say that ideally what is written down should be enuf to recapture the lecture. But without notable devices to shorten or replace natural language, it can hardly be done.

I had a professor in graduate school who tried to write on the chalkboard everything that what would be printed in a book. He virtually succeeded. Everybody had writer’s cramp constantly. The professor wrote like crazy = extremely rapidly. The students could hardly keep up either in copying or thinking. And I don’t believe that understanding was well served.

On the other extreme I have had professors who hardly wrote anything on the board besides a little symbolism and this situation was much worse for the students' understanding.

Mathematics must be written. Just to talk mathematics without writing anything down requires two experts - one to speak and one to listen. In teaching mathematics it is absolutely necessary to write (or show writing) while talking. The student in a lecture class must both listen and write. It is generally easier to write mathematics than to talk mathematics. It is a rare student who can ask a well-framed question in mathematics. Usually, the teacher must interpret the question (that is nonsense if taken literally) and then answer.

In my own lecturing I devised time-saving space-saving procedures that allowed me to write on the board a reasonably full account in reasonable time that the students could copy in reasonable time. Some of 'my style' comes from those days but I have added much more since then.

For an inspiriting discourse on one of the topics of the biography, history, pedagogy, psychology, sociology, etc, of mathematics there is no equivalent substitute for the living voice of a magisterial broad-gauge many-sided scholar, speaking in well-turned sentences and pear-shaped tones. Audio/video tapes and printed transcriptions and Web sites are possible optimal optional opportunities to learn and be moved. A compilation of facts and central features could take the form of a table say. Visual supportive material as paintings, portraits, photographs, etc, must be reproduced unaltered.

☐ what the professor said

The professor of one of my upper-division mathematics courses once said in class something like the following:

‘When you write up an exercise, your work should have the form of an essay for an English course. It should have a central theme. It should be divided into paragraphs. The paragraphs should be divided into sentences. Each sentence should be complete with a subject and a predicate, and it should be fully punctuated, beginning with a capital letter and ending with a period.’

This exhortation summarizes the traditional way of expressing mathematics in print or ready to print. I am sure that it is a rare thing for a mathematics teacher to expect the English-essay form on all homework and tests or even some.

No natural language such as English is designed for the sake of mathematics exposition. I submit that the form of an English essay is not invariably the ideal form of mathematics exposition. What is better and when? In my opinion the full answer to this question is still in the future. It will take time and hard work to find out. I would suggest that various forms should be available for various contexts.

School geometry textbooks based on Euclid's 'Elements' have tried for long years to produce a standardized relatively simple format for geometric theorems and proofs. It is doubtful that complete success has been attained. And whatever format was used in a particular geometry book likely did not lend itself to use in other fields.

Many mathematics teachers say that the 'idea of proof' is the greatest stumbling block to mathematics students. It is possible that one reason for this situation is the lack of adequate standardized formats for mathematical proof.

☐ what the student said

A student in one of my calculus courses once said to me: ‘I envy you. I work hard to see a little bit of calculus at a time. But you must see the entire course all at once.’

‘Yes’, I replied, ‘the best way to learn mathematics is a little bit at a time over a long period of time. Mathematics is so concentrated that is takes a relatively long time to mentally digest a relatively small amount. And then after a while, something wonderful happens. Everything begins to take shape and to fall into place, and then you see it all together as a unity. However, this awareness of oneness, this concept of unification is not something that can be taught in a few minutes or at all. It is something that each person must experience individually.’

Math Art

A One Act Play
by Walter Gottschalk

Characters

• Ms Lois Carmen de Nominator,
a student in Calculus 2

• Professor Minnie Max,
Professor of Mathematics
at Wespoofem University,
teacher of Lois in Calculus 2,
known to her students
as M&M or just Min

(The curtain rises on a University hallway. The audience sees a free-standing bulletin board on which are pinned several sheets of paper and at the top of which the audience can read MATH ART in large letters. Lois and Min are standing on either side of the board and looking at it.)

Lois. Professor, what is ‘Math Art’?

Min. In full, Mathematics Art. To be distinguished from Mathematical Art which is art with mathematical allusions only. For example, the later work of the Dutch painter Mondrian in the first half of the twentieth century. He painted patterns of irregular rectangular grids with thick black horizontal and vertical lines and used bright primary colors for enclosed areas. Those paintings suggest geometric notions or use geometric notions but they do not transmit any mathematical message or theorem. I would call that mathematical art. But mathematics art, as I use the term, is mathematics itself that may be considered so beautiful in both content and form that it may be called art. You could say that some mathematical formulas and diagrams and models, either two-dimensional or three-dimensional, or some combinations of them, are beautiful just to look at. If you understand them, so much the better.

Lois (pointing to a sheet on the board). The title of this particular Math Art is “The Telltale (spelling) T-A-B-L-E-A-U of Calculus.’ What does that word mean?

Min. Pronounced tab-LOH. It's now a word in the English language that was borrowed from the French language and means 'picture.' The word 'telltale' means 'informing, bearing a tale, telling a story, telling a tale.' It's a poetic word that has lots of that musical letter el. Edgar Allan Poe was very fond of using els in his poetry. Also remember his short story 'The Tell-Tale Heart.'

Lois. If 'tableau' means 'picture', why don't they just say 'picture'?

Min. Well, the word 'tableau' has a connotation of 'artistic graphic striking description or representration.' It's what the French call 'le mot juste' meaning 'the exact word.' Also the phrase 'The Telltale Tableau of Calculus' has six els and four tees as well as other repetitions. Why shouldn't mathematics sound good too?

Lois. Is this an assignment that we have to memorize?

Min. You are invited to look at it for a few minutes. Not just for fun but also for practical profit. You know all the individual facts in this display. But this arrangement may give a coherence that is not only pretty but also may help in understanding calculus. The tableau contains a good fraction of the content of first-year Calculus. It may be a gestalt that organizes facts somewhat like the mind does automatically.

Lois. Is this all you have?

Min. No, this is just a sample from a larger packet.

Lois. Could I see it?

Min. I thought you'd never ask. Here it is. (Handing Lois a clear plastic envelope that contains a packet.)

CURTAIN